Exploring the biomechanics of digitigrade stilts using techniques from legged robotics

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Abstract. Stilts are a popular technique for transforming the proportions of a performer's body into those of a fantasy creature, however, they are not typically designed to complement the biomechanics of the human body and therefore require specialized training and exhausting effort to wear. This paper documents the first steps in the design of digitigrade stilts using techniques from biology-inspired robotics, including trajectory optimization and rapid protoyping. We will investigate the effect of digitigrade stilts of different lengths on walking gait, and design supports inspired by ostrich toes to counteract the difficulties of standing in these stilts.

1 Introduction

Digitigrade stilts are a form of prosthetic stilt that is used in film, theatre and *cosplay* ("costumed play") to give the wearer's legs the appearance of the *digitigrade* (toe-walking) posture seen in the hindlegs of mammalian quadrupeds or birds, rather than the *plantigrade* (heel-walking) posture found in humans. This is helpful for portraying characters that combine human and animal anatomy; for example, the werewolves in the 2003 film *Underworld* were created by actors wearing digitigrade stilts. Fig. 1 compares a human leg in these stilts to the leg of an ostrich to show how this illusion is created.

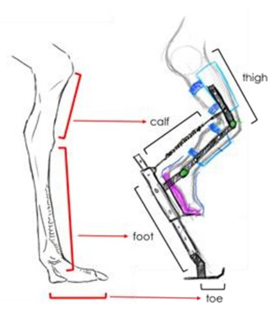

Fig. 1. Comparison between the segments of an ostrich leg, and a leg in digitigrade stilts.

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The only commercially available digitigrade stilts are *Digilegs* by Area 51 Design, a prop and costume design company based in the United Kingdom. These are very expensive (retailing for over of R20 000, excluding shipping), leading many aspiring fauns, freaks and *furries* to build their own stilts instead, as evidenced by the widespread availability of digitigrade stilt-making tutorials available online.

The stilts are often heavy and cumbersome (even Digilegs weigh in at 6 kilograms per stilt) and can also be difficult to balance on when stationary due to a small support base, requiring the wearer to step constantly from side to side. For these reasons, they are generally limited to brief use on hard, even terrain,

As one might expect, academic resources regarding the design of digitigrade stilts are non-existent. While several studies have considered the biomechanics of stilt walking, with the intention of informing improved stilt designs, these have focussed on functional stilt use in occupations such as construction and farming. The stilts used in these fields are very different from digitigrade silts: they are typically much taller, keep the foot flat rather than at an angle, and have a wide support base that enables stationary standing. There is therefore a gap in the literature for similar biomechanical studies that could help would-be digitigrade stilt makers with their designs.

This would seem to be a gap that scientists from the field of legged robotics are well-positioned to fill. Hence, the purpose of this paper: to use knowledge and techniques from this field to provide a scientific foundation that creatives can use to guide their work and potentially build safer, more comfortable costumes. The paper has an anthology format, documenting work on two preliminary projects towards a new stilt design, namely:

(1) An investigation into the effect of digitigrade stilts of increasing length on various gait parameters using trajectory optimization, with the aim of establishing the effect of stilt length on comfortable and efficient walking. This section contributes novel insights into how limb morphology affects the energetics of bipedal, digitigrade gait, using a simplified model that allows them to be generalized to other subjects with roughly humanoid proportions. They might therefore be useful in other applications, such as the design of bipedal robots.

(2) The rapid prototyping of an ostrich-inspired toe with the aim of allowing wearers to balance while stationary, while preserving a relatively small footprint. The contribution of this section is an open-source foot design that can be 3D printed.

The paper will begin with a brief discussion of the biomechanics of digitigrade stilts, to establish the postural *pain points* that must be considered throughout the work.

2 The Biomechanics of digitigrade stilts

Digitigrade stilts create the appearance of a digitigrade leg by elongating the foot. To understand how this affects posture and gait, this section will consider the scientific literature regarding leg anatomy in animals, in combination with descriptions of digitigrade stilt-walking technique obtained from experienced performers.

Metatarsal length is one of the primary morphological identifiers of digitigrade posture in mammals [1], with digitigrade animals tending to have much longer feet than their plantigrade counterparts. In digitigrade bipeds – namely, terrestrial birds – the length from ankle to the joint of flexion in the foot (usually, the metatarso-phalangeal joint) is close to the length of the tibia in many species [2], which gives the impression of a backward-bending leg when combined with the relatively short and less mobile femur. Longer stilts, with a length approaching that of the shank, therefore create a stronger resemblance to animal limbs, and may be more desirable as a result. However, the visual effect needs to be balanced against their effects on standing posture and gait parameters.

For comfort, three main factors need to be considered:

1. the ability of the wearer to keep the joints close to their normal neutral positions, that is, knees not constantly bent and ankles not constantly plantarflexed into a *pointe* position;
2. the loading of the joints, including the torques required to drive locomotion, and the torque exerted on the ankle due to the ground reaction forces at the toe; and
3. the overall efficiency of gait.

The effects of a longer foot on standing posture are obvious from simple geometry: because it tends to shift the support base forwards, for the centre of mass (COM) to remain stacked above the centre of pressure (COP), the wearer must either point the foot, bend the knees, lean forwards with the upper body, or some combination thereof, as shown in part B of Figure 2. While this may lend a more "creature-like" demeanour to the performer, it reduces the duration of wearability, as this position - reminiscent of an isometric squat with the heels raised – places unpleasant torque loads on the leg joints and lower back far beyond those of normal standing. Attempting to relieve the quadriceps by straightening the knees increases the load on the ankles, due to the long lever arm over which the ground reaction forces act in the toe-supported stance.

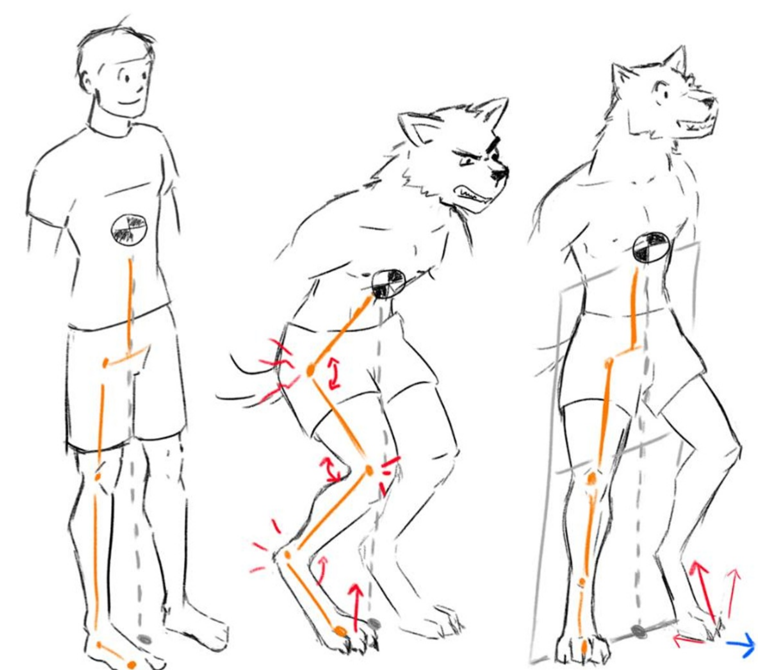

Fig. 2. Standing posture in a normal human (A) compared to a subject combining otherwise similar proportions with elongated feet and digitigrade legs, with feet parallel (B) vs. feet turned out (C). In the parallel stance, the longer foot requires a tiring squatted posture to keep the centre of mass above the base of support, while the turned out stance allows a more upright pose.

A turned out stance (think: "horse stance" in yoga) might be easier to maintain, as it effectively decreases the forward projection of the COP in the sagittal plane, allowing for a more upright posture. This does little to relieve the ankles, however, as, when the knees are more extended, the trade-off there is between unnatural pointe position that reduces the moment caused by the normal force, but uncomfortably loads the balls of the feet, and a lower-heeled posture that allows the ankles to rest in a more natural position, but increases the lever arm of the normal moment. A technique that can reduce the load on the ankles is driving the toes forward, as this leads to a larger static friction force pointing back inwards, which rotates the ground reaction force vector to be more parallel to the foot, thereby reducing the torque experienced at the ankle. Part C of Figure 2 illustrates these changes.

The effects of stilt length on gait are more complex. Digitigrade legs are thought to be an evolutionary adaptation for efficient high-speed running, conveying a longer stride length and mechanical advantage in favour of rapid movement over power transmission [1]. This is also supported by an exploratory study into the optimization of bipedal robot legs for running, which found that bird-inspired backward-bending legs tended to result in a lower cost of transport across a wide space of morphologies, due to an increased ability to push the body upwards and forwards simultaneously during a stride [3]. All known cursorial bipeds are digitigrade, making adaptations for digitigrade posture and adaptations for cursoriality difficult to distinguish from one another [1]. By contrast, humans are adapted to be efficient long-distance walkers, with our anatomy more suited to facilitating the inverted-pendulum-like model of kinetic and potential energy associated with walking, over the spring-like model of running [1,4,5]. It is thus reasonable to assume that longer stilts, which approach the proportions of a cursorial animal, would negatively impact comfortable and efficient walking.

Another challenge of digitigrade stilt-walking is that the contacting area of the toe is generally desired to be much smaller than that of a human foot. Partly, this is for aesthetic reasons, as the smaller toe better imitates the proportions of an animal's paw, but it is also to better enable rolling over the toe during walking, in the absence of an articulated joint between the toe and foot. In some cases, such as the test stilts used later in this paper, the stilts terminate in a rounded point, similar to the feet of quadrupedal robots such as the Unitree *Go 1*, or Boston Dynamics *Spot.* This does, however, make it difficult for wearers to balance, requiring them to make constant readjusting steps when standing. Balance difficulties are also a major obstacle for new stilt-walkers learning the activity [REF].

3 The effect of increased stilt length on gait

3.1 Background: The use of trajectory optimization for biomechanical simulation

Trajectory optimization is a simultaneous simulation method typically applied in robotics for motion planning, where it enables both the state and control trajectories for an unknown motion to be obtained such that a set of constraints is satisfied, and some objective function is minimized [6]. This is also useful for biomechanics, as the framework of optimality is a useful one for navigating the immense redundancy and complexity of locomotion in nature. Organisms can move in an infinite variety of ways, but just a few tend to emerge in practice, so it stands to reason that these are optimal in some way.

When applied to simple *template* models [7], or highly generalized models [8], trajectory optimization can give broad insights about locomotion across different scales and morphologies. On the other end of the spectrum, *predictive simulation* software such as OpenSim Moco [9] allows optimizations to be performed on detailed, realistic musculoskeletal models, so the likely effects of injuries, disorders or other physiological changes can be predicted. Comparison between the observed behaviour of an organism and the trajectories obtained with different objective functions can be used to identify the objective driving that organism's locomotion, a method called *inverse optimal control* [10].

Trajectory optimization is also a powerful tool for comparing morphologies without the confounding variables that would be present in experimental data. For example, it has been used to quantify the contribution of stabilization with the arms to manoeuvrability in bipeds, by comparing minimum-distance stopping trajectories initiated from identical initial conditions within the sprinting gait cycle in models with and without moveable arms [11]. This is also valuable for robot design: the aforementioned study comparing efficient running with forward- and backward-bending legs [3] used trajectory optimization over a large field of randomized models as its core method, while similar studies have been conducted to investigate the most effective leg [12] and spine [13] configurations for manoeuvrable quadrupedal robots. It has also been used to select the parameters of robots more directly, such as determining the optimal leg segment lengths for a bipedal robot for efficient walking [14].

The use of trajectory optimization in this paper is akin to both its preceding applications in biomechanical studies and robot design: we will use it to generate energy-optimal trajectories for the stance phase of walking with different foot lengths, and compare these to evaluate the effects of this parameter on gait.

3.2 Trajectory optimization method

The trajectory optimization was done using the direct collocation method [6], where the simulation of the dynamics and integration of the system's states from one discrete point in time to the next are coded as constraints in a nonlinear optimization problem (NLP). The NLP was written in the open-source Python optimization toolbox, Pyomo [15], and solved using the general-purpose interior point solver, IPOPT [16] with the Harwell linear solver ma97 [17].

To simplify modelling, only motion in the sagittal plane was considered. It was assumed that the toe would be short in comparison to the other leg segments, so it could be represented as a single point, and that it would remain fixed to the ground without sliding. The motion of the arms and swing leg were neglected, and the upper body was constrained to remain perfectly upright throughout the movement. This resulted in a quadruple inverted pendulum model, with three degrees of freedom: the angles of the ankle (θ_a), knee (θ_k) and hip joints (θ_h). This is shown in Figure 3. While these angles were used to bound the ranges of motion of these joints to within realistic limits, and in the subsequent analysis of the trajectories, the absolute angles of the leg segments with respect to the world frame vertical axis ($\theta_1, \theta_2, \theta_3$) were used to formulate the equations of motion, as this results in a more computationally efficient NLP [18].

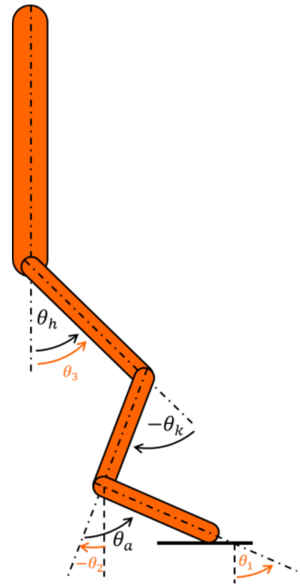

Fig. 3. Planar four-segment rigid-body model of walking stance used in trajectory optimizations.

The lengths, mass distributions and moments of inertia were based on an adult human male [19]. The mass of the torso incorporated the masses of the arms and the swing leg, with the vertical position of the COM and moment of inertia calculated with the arms down at the sides and stance leg straight down with the foot in a neutral position. Joint ranges of motion and torque limits were derived from typical values measured in healthy humans [19].

Another key assumption was that the stilts will ultimately be made of a light material such as aluminium or carbon fibre, so the mass of the extension will be relatively small in comparison to the foot itself. The mass of the foot therefore remains the same as the baseline human foot regardless of length, with the same COM position with respect to the ankle joint. The moment of inertia was, however, increased, by scaling the radius of gyration obtained from the literature according to the increased length of the foot. This might not be the most realistic model, but the intention was to focus the analysis primarily on the gait changes related to the geometry of the segments, in an attempt to gain insights that are less dependent on the mass of the stilts.

The trajectories were discretized into 100 timesteps (termed *nodes),* with the NLP then consisting of the following constraints:

1. equations of motion relating the state variables at each node,
2. integration constraints relating the state variables at each node $n > 1$ to its derivative and the state at the preceding node, according to an implicit Euler formulation, and
3. symmetry constraints requiring the height of the hip to be the same at the start and end of stance, and the contacting toe to be centred beneath the horizontal trajectory of the hip. The translational velocity of the hip was also required to be the same.

The length of the stride was related to its duration by the *Froude number* – a dimensionless quantity roughly capturing the ratio of centripetal force to gravitational force during walking, which is used to compare walking in animals across different scales and morphologies. The Froude number Fr is calculated effective leg length l, (in this case, the extended length of all three leg segments), stride-averaged velocity v and gravitational acceleration $g = 9.81 \text{ m/s}^2$ according to the formula

$$Fr = \frac{v^2}{gl}. \quad (1)$$

The stride-averaged velocity, and hence, the stride length, were calculated for different Froude numbers, stance durations and leg lengths (due to varying foot lengths). The Froude numbers considered were 0.25, a value where walking tends to be most efficient, and 0.5, where the transition to running typically occurs, while the durations considered were 0.3, 0.5 and 0.7 seconds. The foot lengths trialled were 20 values evenly spaced between the length of the foot and the length of the shank. These parameters are summarized in Table 1. For each combination of parameters, 10 random seeds were used to initialize the solver, resulting in multiple local minima being found. The lowest one was then selected for analysis.

Table 2. Simulation parameters

Parameter	**Values**
Froude number Fr	0.25, 0.5
Stance time T [s]	0.3, 0.5, 0.7
Foot length l_f [m]	$l_1 + \frac{l_2 - l_1}{20} k \; \forall \; k = 1,2, \ldots 20$ where l_1 is the natural foot length and l_2 is the length of the shank

The objective minimized was the mechanical work done over the stride, which, for a stride of fixed velocity and length, is equivalent to minimizing the cost of transport. The objective function was the sum of the squared products of joint velocity and joint torque at all joints over all nodes.

3.1 Trajectory optimization results

Fig, 4 shows the cost of transport of the optimal trajectories for all test parameters.

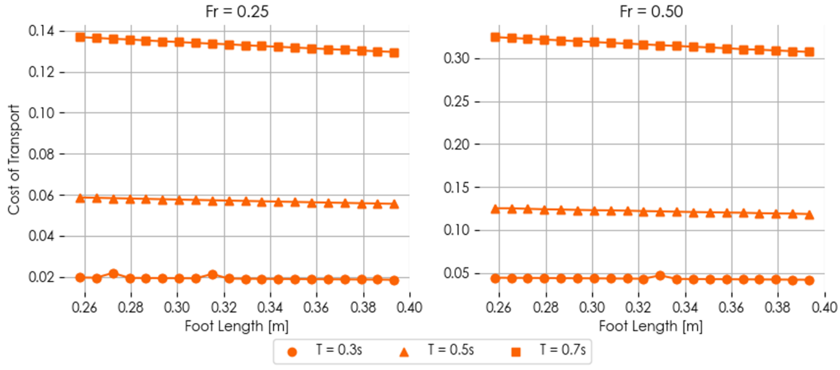

Fig. 4. Cost of transport of optimal stance trajectories for all test parameters.

Perhaps surprisingly, the cost of transport tends to remain similar or even decrease as the length of the stilt increases. This is likely a figment of the assumption that the stilts increase negligibly in weight as they increase in length, as this would result in the model gaining the advantage of the longer stride length without the cost of a much heavier, longer leg with greater inertia. This is not a realistic result, but it does motivate strongly for the selection of as light a material for the stilts as possible, as it indicates that longer stilts do not necessarily impose geometric constraints on the model's ability to walk efficiently if the additional weight added is minimal.

Indeed, visualizing the optimal steps (Figure 5) shows that the length of the stilt actually has almost no effect on the motion of the legs. Even for the longest stride duration, which exhibited the largest cost of transport, the motions are almost identical whether the foot is closer to its human length, or the length of the shank. For a Froude number of 0.25, the motion converges on a near-perfect representation of the inverted pendulum model of passive walking [5], with the length of the leg remaining close to constant, and the hip tracing out a roughly symmetrical arc over the point of support. At the higher Froude number of 0.5, the steps become cumbersomely long, and the trajectory of the hip no longer resembles the ideal pendular behaviour, validating the large increase in the cost of transport.

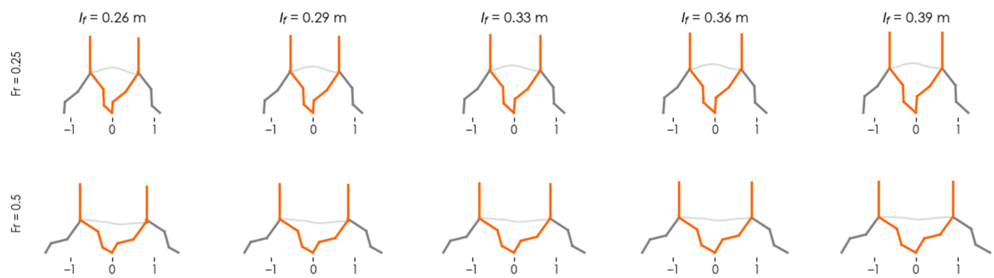

Fig. 5. Illustration of minimal stance trajectories at different foot lengths for stance duration T = 0.7 s. The first column is equivalent to the normal foot length, while the final column has foot length equal to the length of the shank.

This is confirmed by examining the joint angle trajectories over the course of the stride (Fig. 6), which shows that there is almost no difference between those of the shortest and longest foot lengths tested. (A better comparison might be between the stilt trajectories and the baseline motion from around 0.1 seconds into the stride, as the shift in the motion of the ankle from plantarflexion to dorsiflexion indicates that this is roughly where the shift in support from the heel to the ball of the foot occurs.)

Comparison with the baseline of normal human gait (obtained from the baseline values in [19]) indicates that the hip moves in a remarkably similar way to its motion without stilts, albeit with more extension at the end of the stance. The basic waveform of the knee motion is also similar to the baseline, and it remains within a similar range of motion, but starts with a deeper bend and ends in a straighter position. The largest difference is in the trajectory of the ankle, where the joint traverses a much smaller range of motion in the stilt wearer model, centred around a point where the ankle is plantarflexed by around 45 degrees, while the baseline motion is approximately symmetrical about the neutral position at 90 degrees.

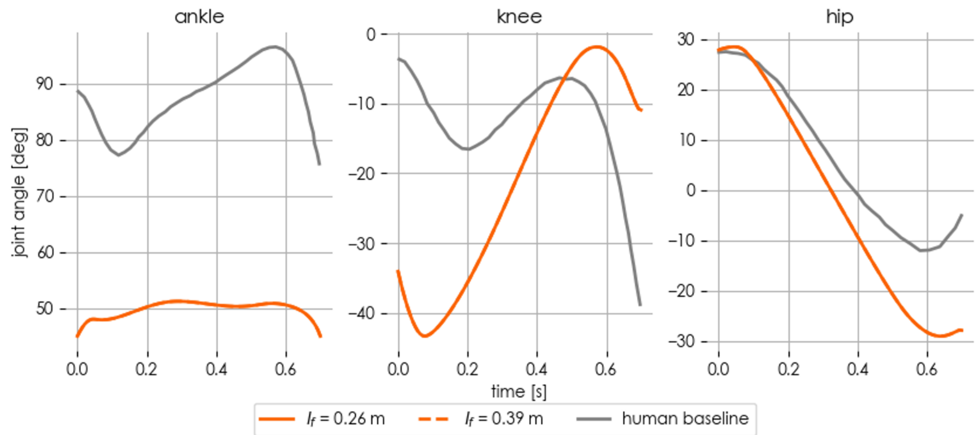

Fig. 6. Trajectories of the joint angles for strides with Froude number Fr = 0.25 and duration T = 0.7 s, compared to the baseline of normal human walking obtained from [19].

The basic traits of normal walking are also retained with respect to the ground reaction forces (GRF), plotted in Fig. 7 (normalized by the mass). The horizontal (x) component has a very similar shape to that of the baseline (again, obtained from [19]), with larger peaks at the start and end of stance, while normal force (y) roughly conserves the double-humped shape characteristic of human walking – also with a much larger initial peak. Although the forces themselves are roughly equivalent regardless of foot length, the longer foot, acting as a longer lever arm, produces a larger GRF moment at the ankle.

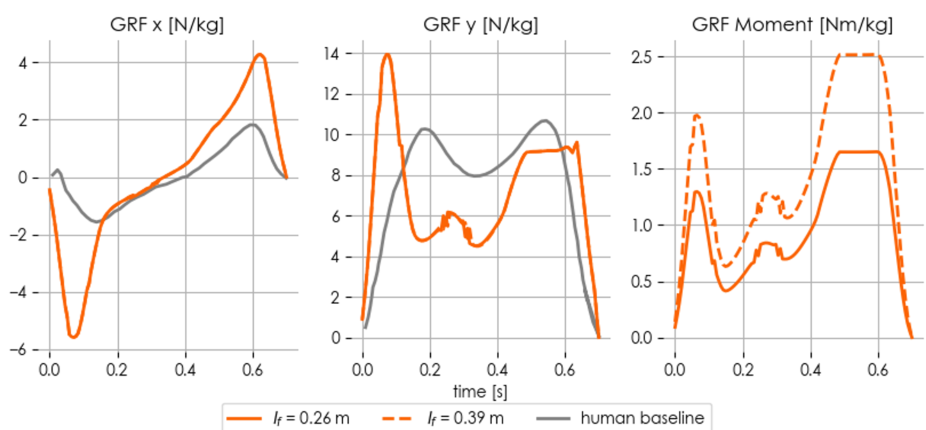

Fig. 7. Ground reaction forces in the horizontal (x) and vertical (y) directions, compared to the baseline of normal human walking obtained from [19], and the ground reaction moment produced at the ankle for strides with Froude number Fr = 0.25 and duration T = 0.7 s. All quantities are normalized by the mass of the model.

3.2 Conclusions from trajectory optimization

The most interesting conclusion from this brief investigation into the biomechanics of digitigrade stilts is that, when only the geometry of the legs is considered, the stilts do not drastically impede the wearer's ability to produce an efficient walking stride. In fact, regardless of length, the most efficient motion is still, in many key respects, very similar to baseline walking.

The greatest pressure point is likely to be the ankle, as this is the one joint forced to operate in a range of motion that drastically differs from its normal trajectory. Due to the large GRF moments associated with elongated feet, this is also likely to be the joint most impacted as the length of the stilt increases. Comfortable stilt designs will therefore need to be mindful of the angle and loading of the ankle. Adding a small wedge between the foot and the stilt, so the wearer's foot is not parallel to the stilt's foot, could be a simple solution that would enable the ankle to operate closer to its natural range of motion. This could also help to alleviate the trade-off between differently uncomfortable ankle positions when standing.

4 Prototyping of ostrich-inspired support

The design of the supporting toe is vital to mitigating the challenges of standing in digitigrade stilts. The aim of this section is to document the biology-inspired design and testing of a 3D-printable toe to be made available as an open resource. The toe was designed with the following objectives in mind:

1. Static stability of the stilts. Ideally, the stilts should be stable enough for the wearer to stand without constantly stepping in place. Besides preventing forward or rearward toppling in the sagittal plane, it is also essential that the supports provide roll stability, as turning the feet outwards (as done in Fig. 3C to improve standing posture) tends to induce supination of the ankles, which could result in severe injury if the stilt then buckles inwards.
2. Traction. It goes without saying that, for basic safety, the stilts must have sufficient grip to resist slipping, but they should also be able to stay in place when the toes are actively pushed forwards, as this results in an orientation of the ground reaction force vector that is less strenuous on the ankles.
3. Avoiding tripping. While longer, wider supports may be more stable, in the absence of an articulated joint, they also make it more difficult to roll over the foot smoothly, clear the ground on liftoff, or, if they extend far behind the toe joint, place the foot down without tripping.

The toe prototypes were tested experimentally, with two basic, qualitative metrics in mind: ease of standing, and ease of walking. The intention is to reduce the barrier to entry for wearers by creating a design that supports both sufficiently that even inexperienced stilt walkers are able to balance and walk unassisted.

The ostrich (*Struthio camelus*) was selected as the model animal for a statically-stable foot design, as this is a digitigrade biped of similar scale to a human. The key feature selected for imitation is the two-toed v-shaped foot, consisting of the long third toe that primarily supports the weight of the body, and the short fourth toe that provides lateral stability. These features are illustrated in Fig. 8.

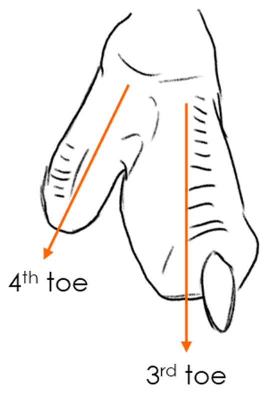

Fig. 8. Key features of ostrich foot selected for imitation in the support for the stilts.

4.1 Rapid prototyping procedure

Five toe designs were tested iteratively before a satisfactory outcome was achieved. These are shown in Fig. 9. The toes were 3D printed from polylactic acid (PLA) thermoplastic, with a 5mm rubber sole glued underneath to provide traction. Each design was tested by attaching them to test stilts (Fig.10A) and having an experienced stilt-walker attempt to walk in them unassisted across a rubber mat and a linoleum floor, following which the design would be altered based on her feedback. These test stilts had a foot length similar to the shank of the stilt tester, and originally had small, rounded toes that could not be balanced on when stationary.

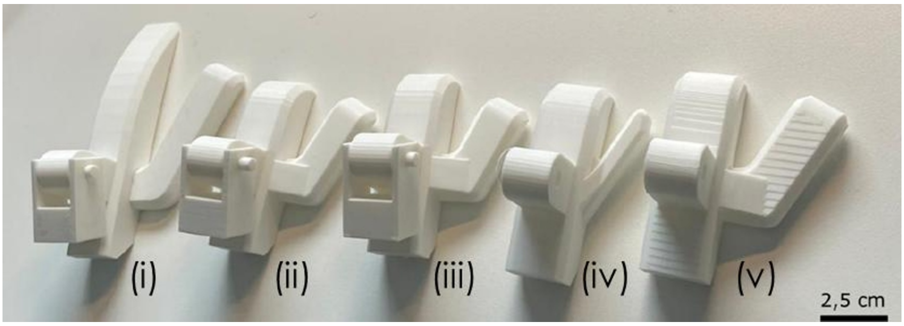

Fig. 9. Ostrich-inspired toe prototypes.

Fig. 10. The test stilts (A). An inexperienced test walker (B) tries the test stilts with their original supports.

As a final test, the stilts were then tested by three subjects with no prior stilt-walking experience (including the other authors), for a 100 m walk on a flat linoleum floor. These testers were also encouraged to try the test stilts with their original point supports as a comparison.

4.2 Toe design results

The first toe design (i) was given the same proportions as the ostrich's toe, but when combined with the rigid connection to the foot, the third toe was found to be too long to roll over easily during weight transfer. This lead to the shorter third toe with a rounded tip in designs ii-v. These subsequent designs refined the length and shape of the lateral fourth toe, while adding length at the rear to prevent toppling backwards. The final toe was sufficiently statically stable to allow the stilts to balance on their own, and to allow wearers of all experience levels to stand still without constantly stepping in place. It was even possible for experienced walkers to balance on one leg! All inexperienced wearers were able to walk unassisted within their first attempts at the 100 m final test walk – something that was not possible even in multiple attempts with the original point foot design.

While the new foot made it possible to stand comfortably in the stilts for relatively long periods of time, and much easier for novice wearers to walk, they were, in some ways, more challenging for experienced wearers to walk with. This is because, even with the shortened, rounded toe, they impede the ability to roll over the toe and therefore limit the wearer's ability to imitate the efficient gaits seen in the point-footed simulation model. In practice, it was only possible to take very short steps, and the knee had to be bent more on touchdown to ensure the foot landed flat. Though it is possible that the foot design could be refined further to better navigate the trade-off between static stability and rolling during locomotion, the next frontier is to consider the possibilities for articulation between the foot and toe.

5 Conclusions

Combining the insights from both preliminary exercises described in this paper, there are several conclusions that can be carried forward into future digitigrade stilt designs:

- Keeping the stilts as light as possible is critical, as from a purely geometric standpoint, even very long stilts do not inhibit the ability to achieve an efficient, humanlike gait.

- The ankles are the pressure point in both standing and walking, as these are required to operate furthest from their natural range of motion in these activities while under large torque loads that are exacerbated by the longer foot. Supporting the foot at a slight incline away from the stilt on a wedge could help relieve ankle discomfort.
- The ostrich-inspired toe is extremely beneficial for static stability as it allows the wearer to balance easily and safely rotate the legs externally to adopt a more comfortable, upright posture.
- As long as the toe is rigidly joined to the foot, there will be a trade-off between stable standing and smooth, efficient walking. Toe articulation should therefore be investigated in future work.

By implementing these adjustments, we hope to develop subsequent protoypes that are more effective at keeping us on our toes, so we can keep unsuspecting festival goers on theirs!

The authors are grateful to Bianka “MechaniCat” Hartenstein, who temporarily donated her wonderfully handcrafted digitigrade stilts to science to serve as the base in the toe design tests, as well as Dr. Janneke Schwaner of the Max Planck Institute for Intelligent Systems, who availed herself as an expert consultant in cursorial bird biomechanics. The rapid prototyping experiments in this paper would not have been possible without the students of the African Robotics Unit who enthusiastically put their bodies and dignity on the line to try out the stilts. Thank you!

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