Resonance assessment of 2-story RC framed structure using linear time history analysis methods

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Abstract. In this paper, we investigate the seismic response of 2D building subjected to harmonic sinusoidal forces at the base with varying frequencies. Linear time history analysis (LTHA) is employed to evaluate the dynamic behavior of 2-story 4-bays reinforced concrete (RC) shear frame to capture the structural response across resonance conditions.

To ensure an accurate assessment of vibration responses, both analytical methods (Modal analysis and Duhamel integration) and numerical approaches (Newmark Beta and Wilson Theta) are employed. Additionally, Finite Element Analysis (FEA) is performed using the SAP2000 software to compare the results. The study reveals that the excitation frequency significantly influences displacement and interstory drift responses. High-frequency excitation does not produce large displacements of the floors.

This work highlights the importance of considering both excitation characteristics and structural properties in seismic design, and it emphasizes the significance of integration analytical and numerical methods in modern seismic engineering practices.

Keywords: RC frame, harmonic force, time history analysis.

1 Introduction

Reinforced concrete frame structures represent the dominant part in the construction sector, however, these structures generally suffer significant damage during severe earthquakes because of the probability of resonance occurring on all or only some floors. The response of the structural frame to harmonic excitations allows us to capture the effect of resonance on the stability of structures, hence its extreme importance. The resonance phenomena may occur when the frequency of the force is approaching that

for natural frequency of the structural system, when it is the case, plastic deformations are developed and can cause catastrophic damage.

Extensive studies were carried out on the behavior of shear frame buildings exposed to harmonic and seismic forces. (A. Tuken 2004) [1] proposed an analytical procedure to determine the lateral displacement of a frame structure under earthquake forces, this approach was then applied to a 3D frame with different heights. The resulting lateral displacements were in good agreement with the SAP2000 results. (A. Tuken 2019) [2] implemented an analytical procedure to compute the lateral displacement of a framed structure subjected to earthquake forces and then applied this procedure to a 3-story shear frame building subjected to a several harmonic loadings applied at the top floor, then he plotted the normalized response amplitude against the frequency ratio, confirming that the structure exhibits its maximum response at the resonant frequency compared to any other load frequency. The amplitude of the vibrations will become very large as the excitation frequency get closer to the natural frequency. (A. Elhelloty 2017) [3] performed a modal analysis to investigate the effect of lateral load resisting systems on the response of buildings under dynamic loads. Three and five stories steel frame buildings were studied for the probability of resonance. A comparative study was conducted to validate the modal and the transient analysis for impulse loading using the ANSYS16 finite element system. He finds that the use of lateral load resisting systems in buildings increases the stiffness and the efficiency of buildings against dynamic loads. (Yizhe Liu et al 2021) [4] conducted a nonlinear dynamic analysis of a three-layer RC frame structure modeled and analyzed basing on the Newmark-β method by means of MATLAB, to capture the variation of the acceleration, displacement, and velocity of the RC frame under the effect of a sinusoidal force. They noted that the application of the force at the top and bottom results in different structural responses in the RC structure. The impact on the structure is more detrimental when the load is applied at the bottom, and the stability of the structure will be greatly reduced as time progresses. (S. A. Pamuji et al 2023) [5] used shaking table to perform a comparative study of sinus earthquake forces and ground motion records on structure behavioral response using linear time history analysis. Considering an 8-story RC frame building to a scale of 1:50. The most significant deviation was caused by the influence of 4.5 Hz harmonic frequency force and by the Kobe earthquake record. The effect of harmonic sine waves is observed to be more significant than the effect of ground motion records.

Urban planning of cities imposes generally a maximum or minimum number of floors for buildings, taking into account the seismicity of the city. In many cities over the world and particularly in Morocco, the majority of residential buildings consist of 1 to 4 floors, which is why in this paper we will examine and discuss the effect of resonance on structural responses of a 2-story 4-bay reinforced concrete frame building under sine wave forces with range of frequencies near to the natural frequencies of the building using analytical and numerical approaches.

2 Modeling-Equation of Motion

2.1 Mathematical Modeling of the building

In order to investigate the effect of resonance on lateral displacements of a building under dynamic loads, we consider a 2D shear frame building with dimension as shown in Fig. 1 Columns and beams have been designed according to the Eurocode-2 [6] taking enlarged static loading into account. The cross-sections of the columns and beams are identical for both stories. The mathematical model "Mass-Spring-Damper System" oscillator is considered to model the building in order to calculate the structural response using analytical procedure and numerical methods; NewMark-Beta (β=1/6 & γ=1/2) and Wilson-Theta (θ=1.4).

The building is composed of reinforced concrete elements, all the columns are 30×30 cm and beams are 25 cm width and 65 cm depth. The material properties of concrete are given as E = 23.5 GPa, ν = 0.2 and ρ = 2500 kg/m^3 for modulus of elasticity, Poisson's ratio and density respectively. Regarding the boundary conditions at the base of the building, the columns are assumed to be clamped (fixed) and the beams are considered rigid. Consequently, the lateral stiffness of each story is calculated using the following formulae:

$$k_i = \sum k_{colunm} = \sum \frac{12EI_{column}}{h_{column}^3}$$

Since the structural damping is relatively small, its effect can be neglected when determining the natural frequencies and modal shapes of structural system. Therefore, in practice, the Eigenvalue problem is solved by applying the same procedure as for undamped structures. The classical damping matrix can be determined by the Rayleigh method by calculating the natural frequencies of the building. Only the first two normal modes are taken in to account in the Rayleigh method:

$$C = a_0 M + a_1 K$$

Where coefficient a_0 and a_1 are obtained by resolving the algebraic equations:

$$\begin{bmatrix} 1/\omega_1 & \omega_1 \\ 1/\omega_2 & \omega_2 \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = 2 \begin{pmatrix} \zeta \\ \zeta \end{pmatrix}$$

The natural frequencies of the structure are 3.34 and 9.97 Hz, mode shapes are (1 ; 1.30) and (1 ; -0.78) determined by solving the Eigenvalue problem. Damping Ratios (ζ) are taken 5% for all modes. Other characteristics of the building studied are summarized in Table I.

Uncoupled modal equations of motion (1) to be solved using the Duhamel integral for each natural mode as a single degree of freedom are written as follows:

$$\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t) \qquad (1)$$

Where D_n are the modal contributions to the displacement, ω_n are the natural frequencies and ζ_n are the damping ratios for the n-th natural mode of vibration. The total response is obtained by combining the response contributions of all the modes (2):

$$u(t) = \sum_1^2 \Gamma_n \phi_n D_n(t) \qquad (2)$$

Where Γ_n and ϕ_n are the modal participation factor and mode shape at the n-th mode.

2.2 Modeling in SAP2000 software

The linear time history analysis calculations were also performed in SAP2000 software for the 2D frame building modeled as shown in Fig. 1 [7]. The Released degrees of freedom for the beam-column joints are the translation along the X-axis and rotation about the Y-axis. We selected "Direct integration" as a solution type using Hilber-Hughes-Taylor method (HHT), with 0.01 second as time step. The HHT-α method introduces a numerical dissipation parameter 'α' to control high-frequency oscillations and reduces unwanted noise in structural dynamic simulations by allowing energy dissipation, this method modifies the standard Newmark equation of motion as follows (3):

$$M\ddot{u}_{t+\Delta t} + (1 + \alpha)C\dot{u}_{t+\Delta t} - \alpha C\dot{u}_t + (1 + \alpha)Ku_{t+\Delta t} - \alpha Ku_t = F_{t+(1+\alpha)\Delta t} \qquad (3)$$

A more negative 'α' increases damping but can also reduce accuracy. Thus, we choose a small negative value to control high-frequency response without losing the precision.

In this study we took α = −0.05 to slightly damp out high-frequency noise without affecting low-frequency response, especially throughout resonance, seeking to depict the effect of the 'α' parameter on the structural response. We note that for α = 0 the Hilber-Hughes-Taylor method is equivalent to Newmark method [8].
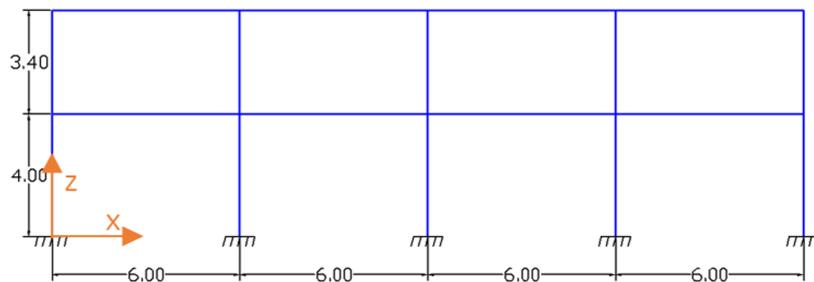

Fig. 1. 2D frame building model.

2.3 Seismic excitation

The loads used in the analysis of the building model are dead loadings, i.e. the mass of the building structure (columns and beams) and dynamic load due to the influence of the sine wave earthquake.

Based on the conclusions of (S.A. Pamuji et al 2023) [5] the dynamic load is approximated by a sinusoidal ground acceleration expressed by $\ddot{u}_{g(t)} = \ddot{u}_{go} \sin(2\pi f t)$ with a range of frequencies f=1.5Hz; 2.5Hz; 3.5Hz; 6.0Hz; 7.5Hz; 10Hz and 12Hz. The

ground acceleration $\ddot{u}_{go} = 1,8g$ is selected from seismic zoning of Morocco as presented in Moroccan Seismic Design code (RPS2000 v2011) [9] with 5 seconds vibration duration. The response shall be calculated for forced system and for the free vibration post-induced structure.

Table 1. Characteristics of the Building

Level	Mass (KN.s²/m)	Stiffness (KN/m)	Story height (m)
Story (1)	14.78	14871.09	4.0
Story (2)	14.09	24215.09	3.4

3 Results and Discussions

The analysis results of the frame building are presented in Fig. 2 to Fig. 5. To ensure the readability of the curves in Fig. 2, we present time history diagrams for only 4 values of frequency. Additionally, we plotted displacement curves for the 3.5 Hz frequency separately to facilitate a comparison of their shapes. The maximum floor displacements and interstory drift ratios of stories are plotted in bar charts as a function of force frequency, distinguishing between the different methods used in this study: Analytical procedure (Modal superposition analysis and Duhamel integral), Newmark-Beta and Wilson-Theta implemented in MATLAB program, and the Hilber-Hughes-Taylor method (HHT-α) using SAP2000 software.

3.1 Lateral displacements of floors

Fig.2 shows the displacement responses of the first and second floor computed using the analytical procedure for different excitation frequencies: 1.5 Hz; 3.5 Hz; 6.0 Hz and 10.0 Hz. Both Steady-State and Transient Responses are considered, and also the response is calculated to the system under forced vibration on the one hand and for the system in free vibration after stopping the excitation on the other hand, in order to identify the moment when the structure's vibration ceases for each frequency. Based on the curves, we can notice the following:

The oscillations cease approximately 5 seconds after the external force is removed, indicating that the 5% structural damping ratio effectively contributed to vibration mitigation in this building. However, it is important to note that Rayleigh damping assumes a linear relationship between damping, mass, and stiffness, which may not accurately represent the damping behavior of complex structures with highly nonlinear characteristics.
At a frequency of 3.5 Hz, the displacement amplitude reaches its peak around the 2nd second. After this point, the maximum displacement gradually decreases until the 5th second due to the influence of the steady-state establishment.
 For high frequencies, the sudden cessation of the applied force (at the 5th second) leads to an increase in the system's displacement at the beginning of free vibration (red and blue curves).

In Fig. 3, the displacement curves corresponding to the 3.5 Hz forcing frequency—which produced the largest displacements—were isolated to highlight the differences between the various methods used throughout the vibration duration. We observe that the curves obtained using the Newmark-Beta, Wilson-Theta and SAP2000 (HHT-α) methods align almost perfectly, whereas the analytical procedure exhibits wider peaks up to the 4th second due to transient response effect. This highlights that the largest displacement peak can occur before the system reaches steady-state conditions. Furthermore, it is observed that the HHT-α method results in a slight increase in amplitude at the onset of free vibrations, which occurs when the applied force suddenly removed. After the 7th second, all curves gradually converge and exhibit nearly identical behavior, indicating stabilization of the system's response—despite the HHT-α method incorporating an additional numerical damping.

Fig.4 shows the maximum values of floor displacements. For both floors, the maximum displacement occurs around 3.5 Hz which is the closest to the fundamental frequency (3.34 Hz). At this frequency, modal analysis method (analytical) predicts the highest displacement values. At the 10 Hz excitation, although the frequency is nearly
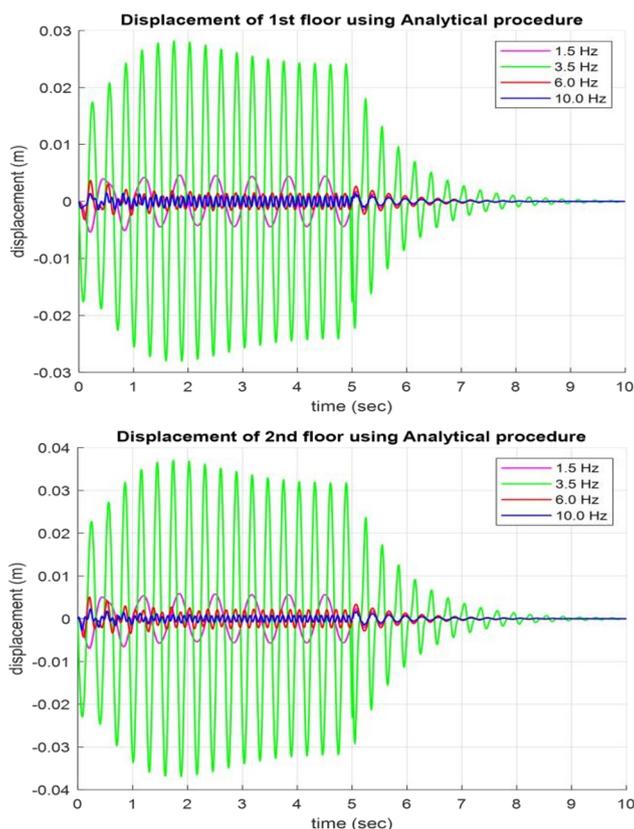
equal to the second mode frequency (9.97 Hz), the resulting displacements are negligible compared to those observed at the fundamental frequency. Thus, the second mode has an insignificant influence on the global response.

Fig. 2. Displacements time history

At both lower and higher frequencies (e.g,1.5 Hz, 6 Hz, etc.), the displacements values are much smaller, mostly less than 1cm. However, it is important to note that low-frequency excitations tend to have a greater impact on the structural response than high-frequency ones. All methods show a similar tendency, with displacements being more significant in the lower frequency range (1.5 Hz to 6 Hz), while beyond 7.5 Hz, the displacements become nearly negligible. Additionally, slight differences in the predicted peak response can be observed among the different numerical methods.

This highlights the significance of resonance effects, which amplify vibrations and result in large displacements. Therefore, it is essential to evaluate the structural stability to ensure that the building can withstand such effects. In the case of this building, the
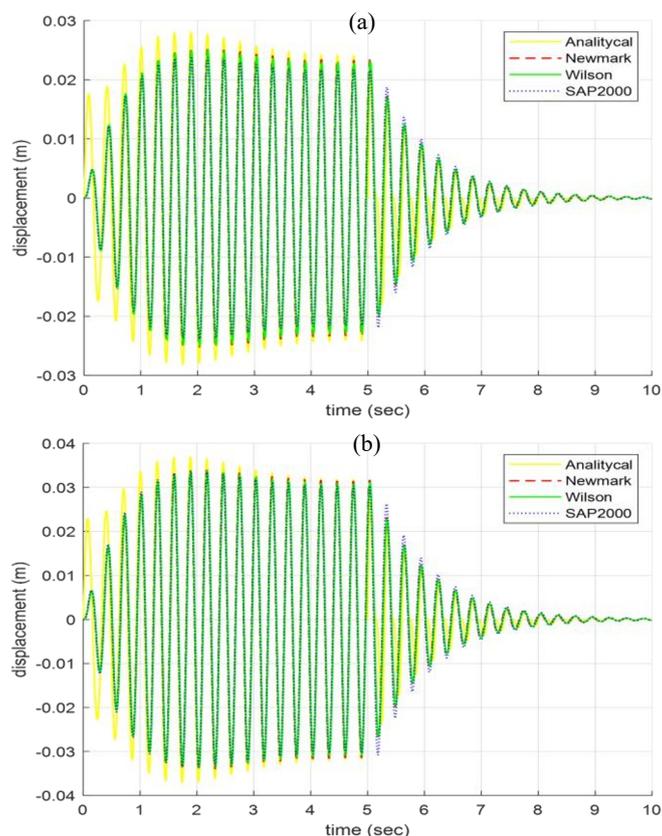
maximum displacement at the frequency of 3.5 Hz slightly exceeds the commonly accepted limit of $0.004 \times H$, where H is the total height of the building [9]. This indicates the need for further evaluation and refinement of the design to improve performance against resonance effects.

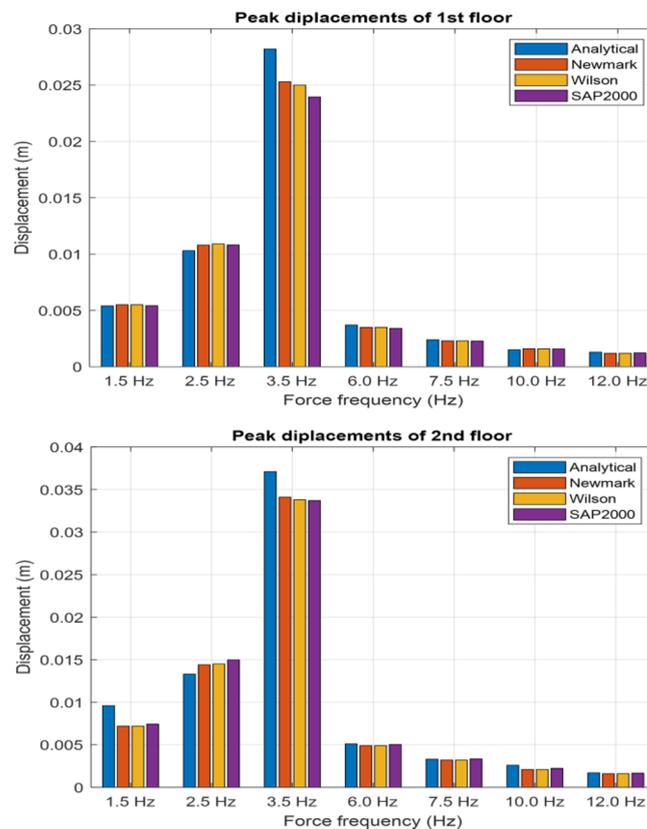**Fig. 3.** Displacement curves at 3.5 Hz frequency: (a) 1st floor (b) 2nd floor.

Fig. 4. Maximum displacements.

Regarding the influence of the numerical dissipation parameter 'α' used in the SAP2000 calculations, it can be stated that a small value of 'α' maintains the accuracy of the response at high frequencies. However, it tends to reduce the displacement values at the resonance frequency, which is particularly noticeable on the 1st floor.

3.2 Lateral interstory drift ratios

The interstory drift ratio (IDR) is defined as the difference in displacement between two consecutive stories divided by the story height. In view of the critical importance

of the IDR parameter, building codes such as Eurocode-8, ATC-40, International Building Code (IBC) and others impose limits on its maximum values. In seismic design, for 'life safety' damage state, the maximum IDR is typically limited by many seismic codes to 0.02 [10] [11].

Fig.5 shows the maximum values of interstory drift ratios as function of force frequencies. From the diagrams, we observe that similarly to floors displacements, the peak drift occurs around 3.5 Hz. Unlike the displacement values, the second natural mode
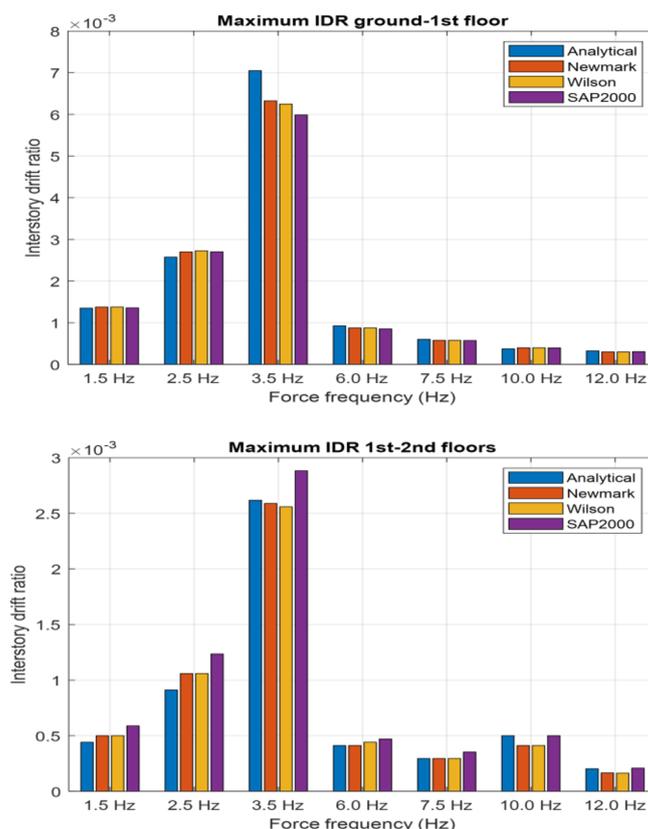

Fig. 5. Maximum interstory drift ratios.

exhibited a more or less significant value of IDR for the second story at force frequency of 10 Hz. The small differences between numerical methods indicate that all methods capture the response well, but some slight variations exist in peak values, particularly in the 2nd story, the response obtained from SAP2000 is mainly the most significant.

Noticing that the maximum values of drifts and displacements occur at the same frequencies, we can propose that for structures with low seismic demand, it is sufficient to validate only the drift values. This simplification can allow for efficient calculations while still ensuring structural performance. Additionally, to further save time in the

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analysis, we can consider only the fundamental mode for buildings of such height, providing that a verification confirms the negligible contribution of higher modes to the overall structural response.

Noting that the maximum IDR at the 3.5 Hz frequency remain well below the allowable limit (0.02) we can state that the building's preliminary design is acceptable for the expected dynamic loads, with slight risks of excessive displacement, particularly during resonance.

4   Conclusions

The main findings of this study can be summarized as follows:

The simplified mass-spring-damper system model is highly effective in predicting the response of an RC building compared to finite element software.
All methods used in this study—modal superposition analysis, Newmark-Beta, Wilson-Thera and Hilber-Hughes-Taylor—showed similar tendencies in displacement and drift responses, with slight variations between methods.
Assessing frame structures for resonance effects must be a crucial phase of structural engineering, especially in preventing soft-story damage.
In dynamic design, it is suitable to employ multiple analytical and numerical methods to achieve more precise estimations of maximum drifts and displacements. This methodology not only enhances the accuracy of structural response prediction but also contributes to optimizing the overall design, guaranteeing greater safety.

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