Jump threshold in a forced and time-delayed Hertzian contact oscillator near the 2-subharmonic resonances

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Abstract. . In this problem we analyze the influence of external feedback time on the threshold in a forced nonlinear Hertzian contact system nearby the sub-harmonic resonance. A perturbation method is performed to approximate vibration amplitudes and their stability near the secondary resonance. The influence of delay state feedback presents in the position, in the velocity or in both cases are studied. The analytical results shown that for appropriate values of the gain and time delay, the frequency amplitudes shift in the direction of high or low frequencies where the softening bistability persists.

Keywords: Hertzian Contact, Time Delay feedback, Perturbation method, Bistabil-ity. Page layout

1 Introduction

In rotating machines, many mechanisms ensuring transformation of motions typically evolve under Hertzian contacts as, for instance, in roller bearing, gears and push-cam systems [1-5]. Such mechanisms evolving in term of preventing large-amplitude vibrations, noise caused by contact jump, requires the mode of contact behavior to be guaranteed when sweeping frequency forward and backward through resonances. This dynamic has been studied for an idealized preloaded and non-sliding dry Hertzian contact considering a one degree of freedom system and using analytical, numeri-cal and experimental approaches [6-8]. It was concluded that the contact loss is typically initiated by jumps in the amplitude near resonances. Thus, to enhance the system performance, the control of vibration response jumps is highly essential. To deal with this problem, different techniques were proposed to avoid such jumps near the primary resonance [9]. The authors propose the first technique

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which consists on a harmonic excitation with fast frequency added to the sinusoidal force applied from above, the second one introduces a fast sinusoidal base displacement, meanwhile we consider the approach with a fast modulated stiffness by sinusoidal expression. The paper proved that the introduced force from above or through base displacement moves vibration amplitude curve to left side, while modulated stiffness displaces the vibration amplitude to right side. Similar study was also carried out near secondary resonances of order 2 [10]. The effect of electromagnetic actuation on the dynamic of a single-sided Hertzian contact forced oscillator is studied near primary and secondary resonances [11]. The influence of time feedback on the vibroimpact threshold in a forced Hertzian contact system was analyzed near the primary resonance considering the cases where the delay is introduced either in the displacement, in the velocity, or in both [12]. Recently, a model combines the delay time and Hertzian contact by taking into account the cutting tool-workpiece interaction, cutting tool geometry and material properties was studied and the Stability Lobes Diagram (SLD) was generated [13].

In the present problem, we analyze the time delay effect on the contact loss of a forced Hertzian contact system near the subharmonic resonance by considering the delay feedback introduced the in deformation and velocity. In section 2 the mathematical model is presented and the modulation equations of frequency-amplitude near the 2-subharmonic resonance is performed using the multiple scales method. The influence of variation in system parameters on the stability regime is reported. To support analytical approximations numerical simulations are conducted. In section 3 we give a conclusion.

2 SUBHARMONIC RESONANCE

Consider the following one degree of freedom system modeling a vertical forced Hertzian contact system under time delay feedback in position and velocity with the mass m attached by a nonlinear restoring force (Johnson, 1979) [14]. A mechanical adoption is shown in Fig.1 and the dynamic equation reads

$$m\ddot{Z} + C_1\dot{Z} + K\,Z^{3/2} = N(1 + \sigma\cos\Omega\,t) + \lambda Z_{T^*} + C_2\dot{Z}_{T^*} \qquad (1)$$

we take $Z_{T^*} = Z(t - T^*)$ and $\dot{Z}_{T^*} = \dot{Z}(t - T^*)$. Here Z is the normal displacement of the rigid mass, C_1 the damping coefficient, K the constant given by the Hertzien theory [14], N the static normal load, σ and Ω are the level of the excitation and its frequency, respectively, while λ and C_2 denote the gains of the delayed states and T^* is the time delay.

The equilibrium position of the contact deformation Z_s is given by $Z_s = \left(\frac{N}{K}\right)^{2/3}$. We use the variable changes as $\omega = \frac{\Omega}{\upsilon}$, $\upsilon^2 = \left(\frac{3K}{2m}\right){Z_s}^{1/2}$, $\eta = \frac{\lambda}{m\upsilon^2}$, $\xi_1 = \frac{C_1}{m\upsilon}$, $\xi_2 = \frac{C_1}{2m\upsilon}$, $x = \frac{3(Z - Z_s)}{2Z_s}$, $\tau = \nu t$ and $T = \nu T^*$, the new adimentionel equation of the dynamic can be written as

$$x'' + 2\xi_1 x' + (1 + \tfrac{2}{3}x)^{3/2} = 1 + \sigma\cos\omega\,\tau + \eta x(\tau - T) + 2\xi_2 x'(\tau - T) + \tfrac{3}{2}\eta \qquad (2)$$

The prime x' denotes differentiation with respect to τ.

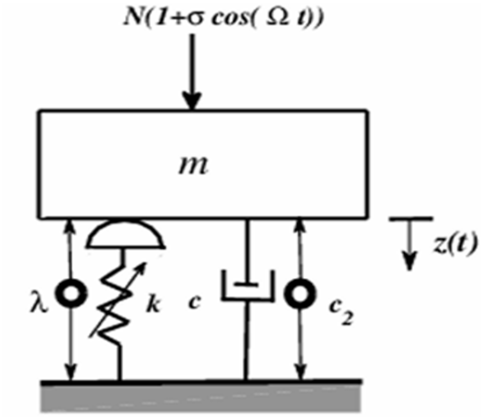

Fig. 1: Schematic description of the model

We only consider the terms of order greater than three in x by development the non-linear deforming force in Taylor series around the static deformation, Eq. (2) becomes

$$x'' + \alpha_1 x' + \omega_0^2 x + \beta x^2 - \gamma x^3 = \sigma \cos \omega \tau + \eta x(\tau - T) + \alpha_2 x'(\tau - T) + \frac{3}{2}\eta \qquad (3)$$

where $\alpha_1 = 2\,\xi_1, \alpha_2 = 2\xi_2, \beta = \frac{1}{6}, \gamma = \frac{1}{54}$ and $G = -\frac{3}{2}\eta$. To explore the vibration amplitude near the 2-subharmonic resonance, we express the resonant condition by introducing a detuning parameter δ according to $\omega_0^2 = \left(\frac{\omega}{2}\right)^2 + \delta$.

The perturbation of the dimensionless parameters of the problem uses a small bookkeeping coefficient μ and scaling the parameters in Eq 3 as $\alpha_1 = \mu\alpha_1$, $\alpha_2 = \mu\alpha_2$, $G = \mu G$, $\beta = \mu\beta$, $\sigma = \mu\sigma$, $\delta = \mu\delta$ and $\gamma = \mu^2\gamma$, yields

$$x'' + \frac{\omega^2}{4}x = \mu(-\alpha_1 x' - \beta x^2 + \sigma \cos \omega \tau - \delta x + \eta x(t - T) + \alpha_2 x'(t - T) - G) + \mu^2 \gamma x^3 \quad (4)$$

Using the multiple scales method [15] a solution to (3) is sought in the form

$$x(t) = x_0(T_0, T_1, T_2) + \mu x_1(T_0, T_1, T_2) + \mu^2 x_2(T_0, T_1, T_2) + 0(\mu^3) \qquad (5)$$

where $T_0 = \tau$, $T_1 = \mu\tau$ and $T_2 = \mu^2\tau$. In terms of the variables T_i , the time derivatives take forms:

$$\frac{d}{d\tau} = D_0 + \mu D_1 + \mu^2 D_2 + O(\mu)^3, \qquad (6\text{-a})$$

$$\frac{d^2}{d\tau^2} = D_0^2 + 2\mu D_{01} + \mu^2 D_1^2 + 2\mu^2 D_{02} + O(\mu)^3 \qquad (6\text{-b})$$

where $D_i^j = \frac{\partial^j}{\partial^j T_i}$.

Substituting (5) into (4) and equating terms of same powers of μ, we obtain the following hierarchy of problems

- Order μ^1 :

$$D_0^2 x_1 + \left(\frac{\omega}{2}\right)^2 x_1 = -(2D_{01} + \alpha_1 D_0 q_0 + \delta)x_0 - \beta x_0^2 - G + \sigma \cos\omega\,\tau + (\eta + \alpha_2 D_0)x_0(\tau - T) \quad (7)$$

- Order μ^2 :

$$D_0^2 x_2 + \left(\frac{\omega}{2}\right)^2 x_2 = -(2D_{01} + \alpha_1 D_0 + \delta)x_1 - (2D_{02} + D_{11} + \alpha_1 D_1)x_0 - 2\beta x_0 x_1 + \gamma x_0^3 + (\eta + \alpha_2 D_0)x_1(\tau - T) + \alpha_2 D_1 x_0(\tau - T) \quad (8)$$

The first approximation of the solution is given by

$$x_0(T_0, T_1, T_2) = R(T_0, T_1, T_2)\cos\left(\frac{\omega}{2}\tau + \theta(T_1, T_2)\right) \quad (9)$$

Hence, substituting (9) into (8) and separating real and imaginary parts, we obtain up to the second order the modulation equations

$$\begin{cases} \frac{dR}{d\tau} = AR - HRsin2\theta \\ R\frac{d\theta}{d\tau} = BR - CR^3 - HRcos2\theta \end{cases} \quad (10)$$

where A, B, C and H are given in Appendix. Eliminating the phase from the system (10), we obtain the following vibration response equation

$$C^2R^4 - 2BCR^2 + A^2 + B^2 - H^2 = 0 \quad (11)$$

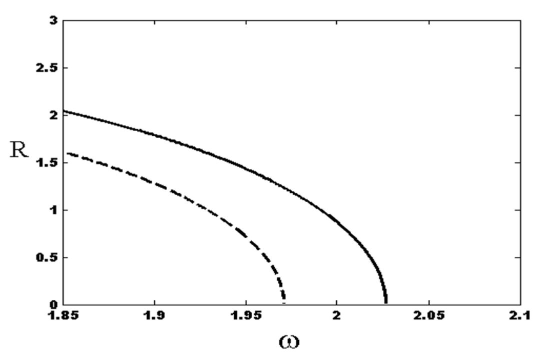

Fig. 2: Vibration amplitude in the absence of time delay ($\eta = 0, \xi_2 = 0$).

Next, the influence of different parameters of the system on vibration amplitude is examined. We fix the parameters $\sigma = 0.5$ and $\xi_1 = 0.0001$ and we analyze the influence of delay state feedback on the frequency amplitude. Figure 2 depicts this response variation near the 2-subharmonic resonance, as given by (11), in the absence of the feedback gains ($\eta = 0, \xi_2 =$

0). The solid line corresponds to the stable solution, while the dashed line corresponds to the unstable one.

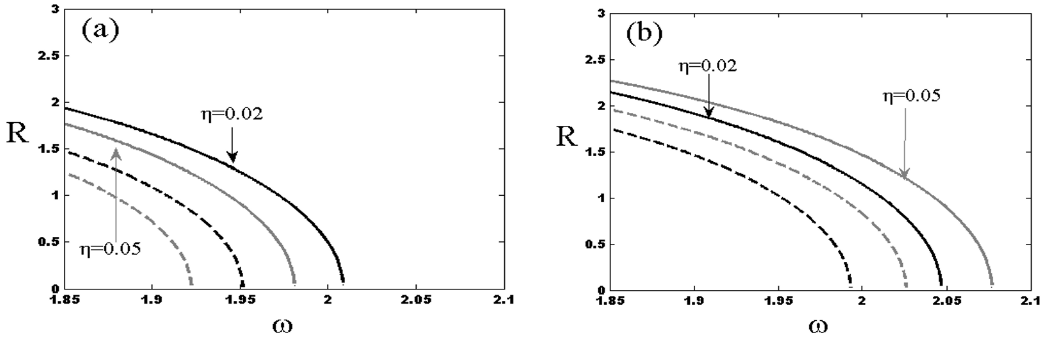

Fig. 3: Vibration amplitude for $\xi_2 = 0$ and for specific values of η; $(a)\ T = 0.1$ and $(b)\ T = 3$.

Figures 3a, b depicts the influence of the delay amplitude in the position ($\eta \neq 0; \xi_2 = 0$) on the vibration response for two different values of the time delay $T = 0.1$ and $T = 3$. These figures show that for $T = 0.1$ an increase of η shifts the vibration response toward lower frequencies (Fig. 3a), whereas for $T = 3$ an increase of η shifts the vibration amplitude right (Fig. 3b). This phenomenon can be clearly seen in Fig. 5 in which the amplitude displacement decreases at T = 0.1 and increases at T = 3.

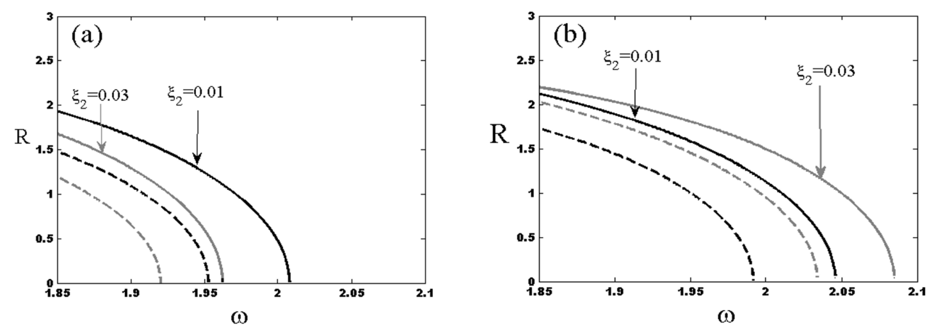

Fig. 4: Vibration amplitude for $\eta = 0$ and specific values of ξ_2; (a) $T = 2$ and (b) $T = 4.5$.

The effect of varying the feedback gain in the velocity only ($\eta = 0; \xi_2 \neq 0$) is depicted in Fig. 4. It can be seen that for $T = 2$, increasing ξ_2 causes the frequency response to shift left (Fig. 4a), while for $T = 4.5$ the shift is to the right (Fig 4 b). This change in the direction of the frequency amplitude shift can be understood by expecting Fig. 6 in which the amplitude decreases for T = 2 and increases for T =4.5.

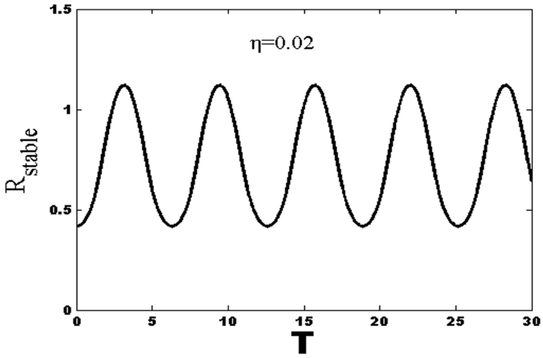

Fig. 5: Variation of the stable amplitude versus T for $\omega = 2$ and $\xi_2 = 0$.

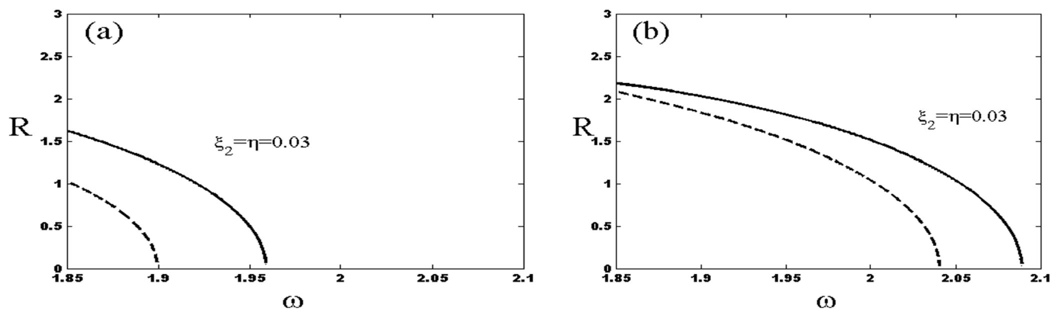

Fig. 6: Vibration amplitude in the presence of η and ξ_2; (a) $T = 1.25$ and (b) for $T = 4$

3 CONCLUSION

The problem of delay impact on the predicting vibroimpact threshold in a Hertzian contact system has been examined near the subharmonic vibration amplitude. The perturbation analysis was used to drive the approximation of the modulation equations and different cases of feedback gains were considered. The results show that, near the 2-subharmonic resonance, increasing the delay gains produces a shift of the frequency amplitude to attend higher or lower frequencies for a given appropriate values of delay feedback, whereas the softening behavior which is a precursor of contact loss persist.

Appendix

$$A = -\left(\frac{\alpha_1}{2} + \frac{\eta}{\omega}\sin\phi - \frac{\alpha_2}{2}\cos\phi\right)$$

$$B = \frac{\delta}{\omega} - \frac{\alpha_2}{2}\sin\phi - \frac{\delta}{\omega}\cos\phi \qquad + \left(\frac{\delta\alpha_2 - \eta\alpha_1}{\omega^2}\right)\sin\phi + \left(\frac{\alpha_1\alpha_2}{2\omega} + \frac{2\eta\delta}{\omega^3}\right)\cos\phi - \frac{\eta^2}{\omega^3} - \frac{\delta^2}{\omega^3} - \left(\frac{\alpha_1^2 + \alpha_2^2}{4\omega}\right) - \frac{\beta G}{\omega^3}$$

$$C = \frac{10\beta^2}{3\omega^3} + \frac{3\gamma}{4\omega} \;, H = \frac{4\beta\sigma}{3\omega^3}, \phi = \frac{\omega T}{2}.$$

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