Simplified Analysis of The Dynamic Longitudinal Track-Bridge Interaction of a Simply Supported Railway Bridges

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Abstract. In the present work, the core idea is to analyse the influence of longitudinal track-bridge interaction (TBI) on the vibratory response of a simply supported (S-S), single-track, skewed bridge beam crossed by moving high-speed trains. The bridge is modeled as a uniform Bernoulli-Euler (B-E) composite beam with two layers connected continuously by horizontal spring and dashpot elements at the slip interface. The skew angle effect is incoporated – refering to a prior model - at both ends of the beam using two rotational spring elements, ensuring that the bridge's behavior is governed sololy by bending modes. By considering the fundamental mode of vibration, two numerical schemes – the Runge-Kutta method (RKM) and the Finite Difference Method (FDM) – are employed to validate the simulation results. The obtained results demonstrate that the system is shown to exhibit dynamic behavior governing by a Duffing-like oscillator. Furthermore, they reveal that even the presence of the bridge's skew angle, the ballast introduces additional stifness and damping. These effects generally to increase the fundamental frequency, in consequence raising the critical speed, while reducing the response amplitude both at resonance and non-resonance situations.

Keywords: Skewed Bridge, Resonance, Acceleration, Longitudinal Track-Bridge Interaction. Page layout

1 Introduction

Today, within railway bridge design the fundamental problem is the vibratory response induced by travelling trains at high speeds, in particular, resonance phenomena. Appearance of this phenomena, in new high speed railways or in conventional lines subject to conversion

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for high speeds, generally can conduct to undesirables consequences such as loss of contact rail/wheel, passenger discomfort, event train derailment, ballast instability, etc [1, 2]. To overcome this problem and for the design of new structures, the Eurocode (CEN), current international standard, establishes an upper limits on the vertical acceleration of the bridge deck, in which a fixed values of 3,5 m⁄s^2 in ballasted track, and of 5 m⁄s^2 in ballastless track are given and recommended [3]. In the past, it is showed that single track, short-to-medium span length bridges (12 - 25m) are more susceptible to be exposed to high vertical acceleration levels, due to their lower mass [2]. However, skewed bridges, are not studied in a detailed manner and the number of publications devoted to this type of structures is rather limited and scarse. Nguyen and Goicolea [4, 5] demonstrated that the dynamic behavior of skewed beam bridge, even experiences an inherently coupled 3D behavior due to the contribution of tridimensionnal modes (bending and torsion), can be assimilated with that of a simplified beam governed only by bending modes, like a simply supported partially clamped beam model is proposed. Ryjáček et al. [10] based on experimental and numerical analyses of a net arch railway bridge of skew angle 43°, revealed the influence of the skew angle and pointed the model's sensitivity to the boundary conditions. Martinez-Rodrigo et al. [6] analyzed, based on a 3D detailed finite element (FE) models including ballast track updated from experimental measurements, the effect of skewness on the dynamic response multi-girder prestressed railway bridges. Nguyen and Goicolea [9] to analyze the dynamic behavior of railway bridges, included skwed ones, under moving high-speed trains, presented a user-friendly software application offering an approche combining efficiency with high accuracy.

In this sense, the main purpose of the present contribution is to explore the simplified model proposed by [4] in order to answer the question of how the longitudinal track-bridge interaction (TBI) can affect the vibratory response of single-track skewed railway bridge structures. The organisation of this paper is as follows: in Section 2 the general theory modelling the problem is preseted. Conclusions regarding the most important factors that alter the behavior of the skewed bridges are presented in Section 3. The principal conclusions of the present investigation are given finally in Section 4.

2 Modelling and Theory

2.1 Problem Formulation

Considering a simply supported, partially clamped bridge model as shown in Fig. 1, and schematizing a simplified model for simply-supported classically damped skewed beam bridge, as described in [4, 5]. The angle of skewness, angle between the abutment and the orthogonal line of the bridge centerline, is denoted α. The Bernoulli-Euler (B-E) beam consists of two layers of length L, upper and lower. The upper scematizes the different components of the ballasted track, whereas the lower one represents the bridge deck, they are based on the same assumptions as for conventional B-E beam without interlayer slip and have the same vertical displacement and rotation. The interface between them is introduced as a continuously distributed longitudinal springs and dashpots schematized as a continuously viscoelastic axial forces Ps and Pd. Subscipts ''s'' and ''d'' denote stiffness and damping respectively.

In this model, the interest is only the vertical behavior of the beam-type structure. Therefore, the transverse bending and torsion modes are not considered, following the assumption adopted in [4] for the simplifed model, and extended here to the context of a composite beam (dual beams).

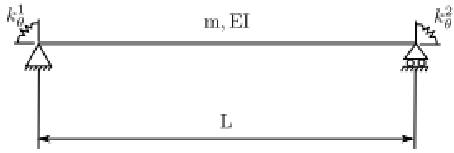

Fig. 1. Simplified model for simply-supported skew bridge [4]

Based on the B-E theory, the partial differential equation (PDE) governing the vertical motion of the skewed bridge beam can be expressed as follows [7]:

$$EI\frac{\partial^4 w}{\partial x^4}+m\frac{\partial^2 w}{\partial t^2}+c_b\frac{\partial w}{\partial t}-P_s\frac{\partial^2 w}{\partial x^2}-P_d\frac{\partial^3 w}{\partial x^2\partial t}=-\sum_{j=1}^{N_v}F_j\left[H\left(t-\frac{d_j}{V}\right)-H\left(t-\frac{d_j+L}{V}\right)\right]\delta\left(x-\left(Vt-d_j\right)\right) \tag{1}$$

where $w(x,t)$ denotes the vertical displacement of the skewed beam, x is the longitudinal coordinates and time t, EI is the total bending stiffness (superposition of that of each beam), GJ is the tortional stiffness, m is the mass per unit length, c_b denotes the structural damping coefficient, F_j is the constant force exerted by the jth load, N_v is the total number of train axles, d_j is the distance separating at time $t=0$ the jth load position from the entrance section of the bridge, V is the constant train speed, $\delta(.)$ denotes the Dirac distribution and $H(.)$ the Heaviside function. The loads are assumed to be concentrated and are represented by means of a Dirac acting at the rails beam points $x_k(t)=Vt-d_k$. Since the bridge behavior in particular is of interest and not that of the trains, the track irregularities and vehicle-bridge interaction effects are overlooked. P_s and P_d are the axial forces located at the slip interface between the ballast layer and the bridge deck, and are defined as follows [8]:

$$P_s=a+b\left|\frac{\partial w}{\partial x}\right|+c\left(\frac{\partial w}{\partial x}\right)^2;\ P_d=d+e\left|\frac{\partial^2 w}{\partial x\partial t}\right| \tag{2}$$

About the stiffness part, it is important to note that a bilinear law is proposed by the EC1 [3] to simulate the non-linear behavior of the horizontal interaction between the ballast and the bridge, but is not integrated here.

As noted before, the beam is simply supported partially clamped by an identical rotational stiffness at both ends of the beam having the following expression [4]:

$$k_\theta^1=k_\theta^2=K_r=\frac{2GJ}{L\cot^2(\alpha)} \tag{3}$$

The associated boundary conditions are defined as:

$$\begin{cases} w(0,t)=w(L,t)=0 \\ EI\dfrac{\partial^2 w(0,t)}{\partial x^2}=K_r\dfrac{\partial w(0,t)}{\partial x} \\ EI\dfrac{\partial^2 w(L,t)}{\partial x^2}=-K_r\dfrac{\partial w(L,t)}{\partial x} \end{cases} \tag{4}$$

As a consequence, based on these defined boundary conditions given by Eq. (4), the mode shape of the simply supported partially clamped beam can be calculated and given as follows:

$$\Phi_i(x)=\gamma_{1i}\left(\cosh\left(\frac{\lambda_i x}{L}\right)-\cos\left(\frac{\lambda_i x}{L}\right)\right)+\gamma_{2i}\sinh\left(\frac{\lambda_i x}{L}\right)+\sin\left(\frac{\lambda_i x}{L}\right) \tag{5}$$

With

$$\gamma_{1i} = \eta \frac{\sinh(\lambda_i) - \sin(\lambda_i)}{2\sinh(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))}; \gamma_{2i} = -\frac{2\sin(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))}{2\sinh(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))}$$

$$K_r = \frac{2GJ}{L\cot^2(\alpha)}; \eta = \frac{4k}{\lambda_i}; k = \frac{K_r L}{4EI} \tag{6}$$

and λ_i can be determined by solving the frequency equation having the following form:

$$2k^2\cosh(\lambda)\cos(\lambda) - 2k^2 + k\lambda\sinh(\lambda)\cos(\lambda) - k\lambda\cosh(\lambda)\sin(\lambda) - \frac{(\lambda)^2}{4}\sinh(\lambda)\sin(\lambda) = 0 \tag{7}$$

Since the studied system is becomes non-linear (Eq. (1)) with the introduction of the longitudinal track-bridge interaction, an analytical solution is complex generally and the numerical methods are still the best choice that can be adopted to found the numerical solution of this non-linear system. In the present investigation, by applying the Galerkin's method, considering only the fundamental mode of vibration, and in virtue of the orthogonality condition, an ordinary differential equation (ODE) is obtained and governed by a Duffing like oscillator as :

$$\begin{aligned}&\ddot{q}_1 + 2\xi_b\omega_1\dot{q}_1 + \omega_1^2 q_1 - \frac{a\int_L \Phi_1''\Phi_1 dx}{m\int_L \Phi_1^2 dx} q_1 - \frac{b\int_L |\Phi_1'|\Phi_1''\Phi_1 dx}{m\int_L \Phi_1^2 dx}|q_1| q_1 - \frac{c\int_L (\Phi_1')^2\Phi_1''\Phi_1 dx}{m\int_L \Phi_1^2 dx} q_1^3 \\ &-\frac{d\int_L \Phi_1''\Phi_1 dx}{m\int_L \Phi_1^2 dx}\dot{q}_1 - \frac{e\int_L |\Phi_1'|\Phi_1''\Phi_1 dx}{m\int_L \Phi_1^2 dx}|\dot{q}_1|\dot{q}_1 = -\sum_{j=1}^{N_v} \frac{F_j\,\Phi_1(Vt - d_j)}{m\int_L \Phi_1^2 dx}\left[H\left(t - \frac{d_j}{V}\right) - H\left(t - \frac{d_j + L}{V}\right)\right]\end{aligned} \tag{8}$$

where q_1 is the generalized displacement of the beam, $\omega_1 = \left(\frac{\beta}{L}\right)^2 \sqrt{\frac{EI}{m}}$ is the fundamental natural frequency of the beam.

Solution of Eq. (8) is obtained then by using Runge-Kutta method (RKM) conjointly with the central difference scheme (FDM) where the initial conditions (displacement and velocity) are all set to zero (at the instant of the entrance of the train to the bridge). All the calculations are performed at the median section of the bridge since the maximum of response is observed here.

3 Numerical Results

In this section, an application of the theory presented previously in Section 2 will be presented here, and all the effectuated calculations process on the hypothesis that the studied structure is a linear time-invariant system. The studied bridge in this investigation is the same one that analysed previously in [4]. It is a single-track beam bridge with a total length L= 24m, mass per unit length m=9774kg/m, Young's modulus E=32GPa, Poisson coefficient ν=0.25, the area moments of inertia I=1.3921$m^4$and J=2.6741m^4, and with skew angle α=10°. The bridge is analyzed dynamically under an HSLM A1 train (high speed load model A1 [3]), or by means of constant moving load model (MLM), at speeds ranging from 100 to 300km/h. The HSLM A1 model consists of 18 intermediate coaches, with a coach length of 18m, an axle force of 170kN, and is chosen for comparion and to stay in line with previously study [4]. Others HSLM A can be, since for bridges with lengths higher than 7m ten types are defined [3], taken into account for studying the effect of geometrical characteristics of trains. The values of the related constants of the longitudinal TBI given in Eq. (2) are [8]: $a = 2.65.10^7$, $b = -3.90.10^{10}$, $c = 1.94.10^{13}$, $d = 1.04.10^5$ and $e = 8.93.10^5$. For the beam modelling, *a Timoshenko beam model was used and similar results have been obtained comparing to the B-E model, but are not presented here. The used time step is 1/1000s.*

3.1 Linear Model

In a first attempt, by disregarding the non-linear effect due to the longitudinal TBI, it is important to compare the anallytically obtained modal characteristics, such as natural frequencies, with a FE simulation (stick model), even this last is time consuming and a computational effort is deployed. Table 1 gathers a comparison between the first five undamped natural frequencies calculated by resolving Eq. (7) and those issues form a FE simulation performed by Nguyen and Goicolea [4]. A good agreement generally between the computed frequencies can be observed, meaning that the analytical calculations can predict with less effort the modal characteristics of the beam in comparison to FE simulation. Linear model here meaning that the studied beam is isolated from any sources of non-linearity and the linear elasticity assumption is adopted.

Table 1. Five first natural frequencies.

Modes	Analytical model	FE model [4]
1	5.878	5.878
2	23.344	23.288
3	52.454	52.455
4	93.152	93.153
5	145.550	145.642

3.2 Non-Linear Model

As preoviously noted, by integrating the longitudinal track-bridge interaction, the system is governed by a Duffing like oscillator Eq. (8), where by retaining the fundamental mode of vibration, a non-linear ordinary differential equation (ODE) with cubic term is obtained. To solve then the resulted ordinary differential equation, two numerical methods are used and compared, FDM and RKM. Fig. 2 highlights the maximum dynamic response, in terms of displacement and acceleration, computed at the median section of the bridge under HSLM-A1 train at 190km/h. It can be clearly observed that the obtained responses by using the two numerical methods are in good correspondence, and the vertical acceleration is still verifying the Eurocode requirement in which is still below 3,5 m/s^2 . In addition, by disregarding the non-linear effects and at the same speed, a good agreement between the present model and the FE simulaton in [4] has been founded.

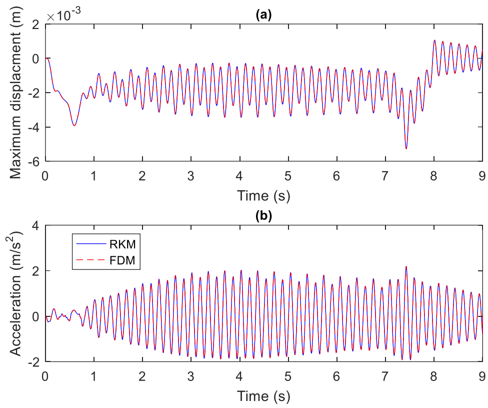

Fig. 2. Mid-span maximum response of the bridge crossed by moving HSLM-A1 train at speed 190km/h: a) displacement, b) acceleration.

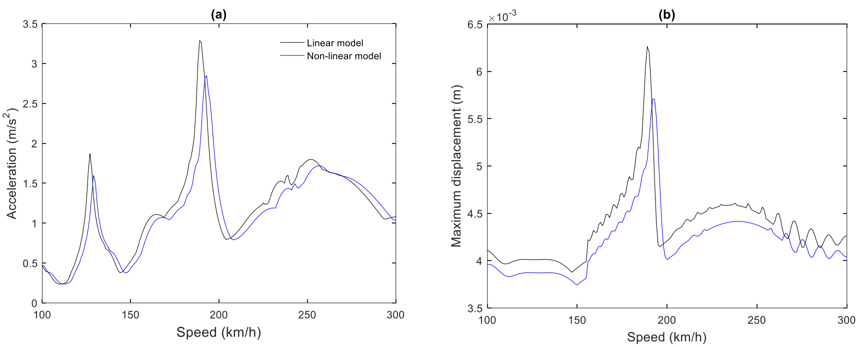

Fig. 3. Effect of the longitudinal TBI on the vibratory response calculated at mid-span section of the bridge versus the HSLM-A1 speed: a) acceleration, b) displacement.

At speeds ranging from 100 to 300km/h with an incremental speed of 1km/h, the absolute maximum response calculated at the position x=L/2 is now presented in Fig. 3. This figure demonstrates a comparison between two cases : without ballast interaction (linear model), and with interaction (non-linear model). Firstly, for the linear model, it is indicated that two resonant peaks appear whose the amplitude is less than $3{,}5\, m/s^2$, and which are in good coincidence with those calculated theoretically and associated to the second and third resonance of the fundamental mode of vibration, the theoretical speeds can be determined as follows [2]:

$$c_r^2 = \frac{f_1 D}{2} = \frac{5.878\times 18}{2} = 190.45\, km/h \; ; c_r^3 = \frac{f_1 D}{3} = \frac{5.878\times 18}{3} = 126.96\, km/h \qquad (9)$$

where D is the characteristics distance between carriages of the articulated train.

Secondly, one can retain that the longitudinal track-bridge interaction, denoted non-linear model in the legend, gives rise to both additional damping and stiffness, in which an increase in the critical speeds associated with a decrease in the peak amplitude can clearly identified when the track is introduced, both in resonance and non-resonance conditions. This result confirms subsequently that the longitudinal slip interface influences the bridge frequency, and this influence is independently of the bridge structure's boundary conditions.

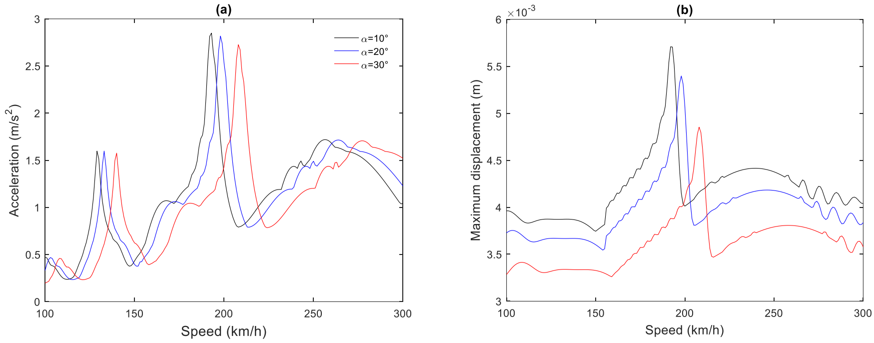

Fig. 4. Effect of skew angle on the vibratory response calculated at mid-span section of the bridge versus the HSLM-A1 speed: a) acceleration, b) displacement.

By concentrating on the non-linear model, Fig. 4 depicts the influence of the skew angle α (variation from 10° to 30°) on the vertical response of the bridge at mid-span section, in terms of vertical displacement and acceleration, under the HSLM A1 train. It is clearly observed that when the skew angle increases the first natural frequency increases, even a non-linear interaction is considered, and in consequence the critical speed in its part. This conclusion is apparently consistent with the result presented in [4, 5] in which the authors studied the bridge adopting at the same time a simplified and general models, but any non-linear effect has been considered. Additionally, it will be important to take into account both the vertical and horizontal in order to understand physically the behavior of the system in its integrity.

4 Conclusion

In the present work, the influence of the longitudinal track-bridge interaction (TBI) on dynamic response of a simply-supported (S-S), single ballasted track, skewed railway bridge is addressed. A simplified model proposed in prior work is adopted, in which the bridge is modeled as a simply supported partially clamped structure comprising two layers: an upper layer representing the ballasted track composed by rails, sleepers and ballat, and a lower one representing the bridge deck. The horizontal slip interface between these layers is modeled using continuously distributed spring and dashpot elements. First, by considering the composite beam as uniform structure and comparing its natural frequencies with those issues from a FE simulation, a good correlation is observed. Secondly, when the non-linear composite effect between the track and the bridge is considered, the system in its integrity is shown to exhibit dynamic behavior governing by a Duffing-like oscilltor. The results indicate that the ballast, even in the presence of a skew angle, introduces additional stiffness and damping. This increases the bridge's natural frequency (thereby raising its critical speed) while reducing the response amplitude under both resonance and non-resonance conditions. Furthermore, increasing the skwe angle appears to amplify the natural frequency of the system.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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