Railway Coupled Ballasted Track-Bridge System: Mode Shape Sensitivities For Damage Detection

Anouar Echchriki1*[0000-1111-2222-3333]**, Mohamed Tahiri[2[0000-0003-1857-2130]] , and Abdelali El-Bakari[1[0000-0002-4196-6640]]

[1]Team Advanced Materials, Structures and Civil Engineering, Lab STIC, National School of Applied Sciences of Tetouan, Abdelmalek Essaadi University, Tetouan 2222, Morocco
[2]Team Mechanics, Mechatronics and Materials, Lab STIC, National School of Applied Sciences of Tetouan, Abdelmalek Essaadi University, Tetouan 2222, Morocco

Abstract. In this study, a theoretical investigation is conducted to analyze how the sensitivity of modal characteristics (mode shapes) in a coupled track-bridge system to structural damage influences the behavior of railway structure. The system under study comprises two straight, parallel, uniform Bernoulli-Euler (B-E) beams, interconnected by uniform vertical springs that model the vertical interaction between the ballasted track and the bridge deck (TBI). For that, a Finite Element model (FE) is implemented, and two methods – Nelson's method and Modal method – are presented for calculating the mode shape sensitivities (derivatives) to enable comparative analysis. Additionally, the Finite Difference Method (FDM) is employed to approximate the curvature of the eigenvector sensitivity, aiming to assess the feasibility of localizing potential damage in both the rail and bridge deck. Results highlight the significance of lower-order mode sensitivity for single damaged-scenarios. However, in cases of multiple damages, challenges arise in identifying precisely the defect locations. Conversely, the curvature of mode-shape sensitivity proves efficiency for damage localization in the coupled system, even when single or multiple damaged elements are present.

Keywords: Railway Coupled System, Mode Shape Sensitivity, Curvature, Damage, Localization.

1 Introduction

In railway tracks, ballasted and ballast-less, either for conventional lines or for new high speed lines, maintenance and structural health monitoring (SHM) are becoming subjects of interest of many civil engineering compagnies and researchers in last years. Generally, the existence of structural damage or crack in a given structure, even if minor or of limited significance, leads to a loose of stiffness and an increase in its flexibility and damping ratio. According to Rytter [1], methods for structural dam-age detection could be classified to four levels: detection, localization, quantification of severity, and finally prevention of the

* Corresponding author: anouar.echchriki@etu.uae.ac.ma

residual life. Theoretically, the damage can introduce a modification of the dynamical characteristics of the structure to be inspected such as, mode shapes, damping ratio and natural frequencies. Consequently, benefiting from these parameters and comparing them between two states, healthy and damaged, could be an efficient tool for damage detection and localization, and to anticipate any defiance in advance. In the literature, several damage indicators based on the evolution of the modal information's structure have been proposed in this sense, this can encompass Modal Assurance Criterion (MAC), Modal Curvature Method (MC), Flexibility Method (FM), Modal Strain Energy Method (MSEM), etc. [2]. Sensitivity-based damage detection methods being one of others methods for damage identification. From the definition, sensitivity analysis aims to understand how a small modification or change in the mechanical properties of a structure (mass or stiffness) can affect its modal parameters (natural frequencies, damping ratios, and mode shapes) [3]. Generally, the eigenvalue sensitivity does not pose a problem, but the eigenvector sensitivity being complex due to the fact that are difficult to be measured accurately. Over the past years, several methods for calculating the derivatives of the eigenvector have been proposed including Modal method [4], Nelson's method [5], the sub-structuring method [6], the modified modal method [7], etc. Recently, Yang and Peng presented a new efficient method to calculate the eigenvector sensitivity; its pertinence is proved compared to the known methods, both for the first and second order sensitivity [8]. Parloo et al. [9] by doing reference to the mode shapes derivatives to a parameter reflecting a change in stiffness or mass of the test-ed structure, they presented a damage index for detection and localization of default. Good results have been obtained, even their proposed approach requires the mode shapes of the structure in two states, healthy and damaged, which is in a practical application some time difficult to obtain such as an information.

In high speed railway bridges, the bridge structure in its integrity is a complex system since several mechanisms of interaction are still existing between its different components, like in track-bridge interaction (between rails, rail-pads and sleepers, ballast and bridge deck), in soil-structure interaction, vehicle-bridge interaction, etc. Added to that, the nonlinear behavior of each element (material, geometrical, etc.) in its part. Obviously, the repetitive nature of moving train axle loads, combined with the effects of aging, can compromise the system's structural integrity over time. For this, search more reliable methods to analyze the structural health of each constitutive element is becoming extremely important, in particular, to ensure the safe opera-tion of the railway system. From the literature, several indirect approaches having the objective to evaluate the evolution of structural health of the railways structures attracted much attention in recent years [10].

In the aforementioned literature and to the authors' knowledge, there is no studies focusing on the prediction of the sensitivities of eigenvectors for damage localization in coupled system schematized as a double beam model. For that, in the present work the interest is to analyze how the sensitivities of the first lower-order mode shapes and their associated curvatures can be explored to estimate the location of damage in the coupled track-bridge system, modeled as a composite structure of two beams connected vertically by a distributed layer of spring elements (dual beam). The organization of this work is as follows: the basis theory for modelling the problem are detailed in Section 2. The main obtained results and discussions are shown in Section 3. Finally, important conclusions extracted from this investigation are given in Section 4.

2 Modelling Theory

2.1 Finite Element Formulation

Fig.1 gives the synthetic model considered for representing the vertical motion of a straight, simply supported (S-S) bridge with ballasted track. The system is composed by two uniform elastic beams undergoing flexure deformation according to Bernoulli-Euler (B-E) hypothesis. The two parallel beams are referred to as upper and lower beam with reference to the gravity acceleration. They are connected by a continuous series of vertical elastic linear springs of stiffness k_f. The upper beam represents the rails with mass per unit length ρA_r and flexural stiffness EI_r, whereas the lower beam represents the bridge structure of mass per unit length ρA_b and flexural stiffness EI_b. Both beams have the same length L and are considered to be simply supported on their ends. The axial deformations are neglected and it is assumed that no horizontal slip takes place between the two beams. 'r', 'b' and 'f' in subscript position refer to the rails, bridge and foundation, respectively.

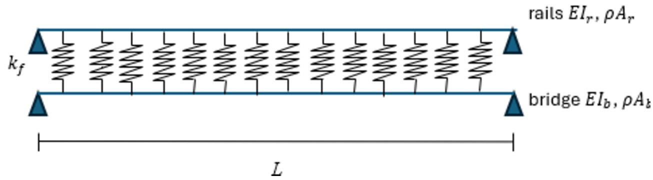

Fig. 1. Bi-dimensional Bernoulli-Euler model of a rail-bridge system

The FE model consists to discretize the coupled beams into n_e identical elements of length l_e as given in Fig. 2, where two nodes per element and two degree of freedom (rotation and vertical displacement) per node. The following notation is used: i and j for the i- and j-end of the rail or bridge element; and v and θ for the vertical displacement and rotation, respectively. Considering a rail-bridge coupled elements with different element lengths is possible, but is out of scope of the present investigation.

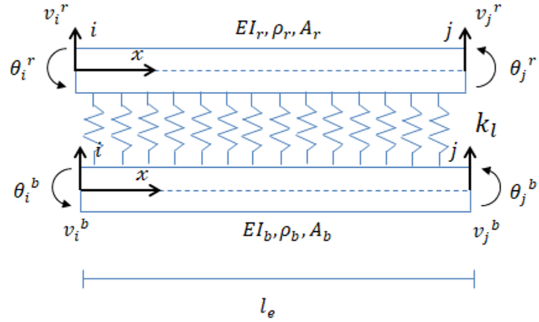

Fig. 2. Schematic track-bridge interaction element

By adopting the B-E theory and using the cubic shape functions, the elemental stiffness and mass matrices of the beams and the elastic foundation can be obtained as follows:

$$\left[m_e^n\right] = \frac{\rho A_n l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 12 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \tag{1}$$

$$\left[k_e^n\right]=\frac{EI_n}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \tag{2}$$

$$\left[k_e^f\right]=\frac{k_f l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 12 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \tag{3}$$

where n=r or b, the subscript f in Eq. (3) denotes foundation.

For the whole coupled beams with rigidly supported conditions, by assembling the elementary matrix, the total mass $[M_t]$ and stiffness $[K_t]$ matrices can be obtained as follows:

$$[K_t]=\begin{bmatrix} [K]_r+[K]_f & -[K]_f \\ -[K]_f & [K]_b+[K]_f \end{bmatrix};\ [M_t]=\begin{bmatrix} [M]_r & [0] \\ [0] & [M]_b \end{bmatrix} \tag{4}$$

where superscripts t, r, b and f designate, respectively, total, rail, bridge and foundation.

Since the main idea behind this study is to explore the modal information of the structure, this can be done by finding the solution of the problem of free vibration of an undamped system of $2n_e+2$ degree of freedom, and we have:

$$\left([K_t]-\omega_k^2[M_t]\right)\{\phi_k\}=0 \tag{5}$$

where ω_k is the kth natural frequencies (eigenvalues) and $\{\phi_k\}$ denotes the mode shapes of the coupled rail-bridge beam (eigenvectors of the coupled system). In what follows, for the eigenvector representation, only the nodal vertical displacements will be retained, and the rotations will be overlooked since the flexural behavior is dominant in this type of structures.

Additionally, due to the fact that the treated problem is considered for a linear undamped symmetric systems and the matrix M is non-singular, the obtained mode shapes (eigenvectors) are all taken as vectors normalized to mass matrix.

2.2 Sensitivity Analysis

By definition, sensitivity refers to how a small change in a structure's properties (e.g. mass or stiffness) affects variations in its dynamic characteristics, particularly mode shapes. As noted previously, mode shapes are the primary parameter of choice for constituting damage localization index. They constitute a reduced scale observation of the structure's real modes, limited to the locations of the measurement points. In this context, only two methods are selected to compute the derivatives of the mode shape with respect to a design parameter of the coupled structure.

2.2.1 Modal Method

As is well-known, due to the fact that is simple for implementation, the modal meth-od remains as a very powerful method and from the definition, it consists to express the derivatives of the concerned eigenvector as a linear combination of all the mode shapes, the mode shape sensitivity is defined as follows [4]:

$$\frac{\partial \phi_i}{\partial p} = \sum_{j=1}^{N} \alpha_j \phi_j \tag{6}$$

where

$$\alpha_j = \begin{cases} \frac{1}{\omega_i^2 - \omega_j^2} \phi_j^T \frac{\partial K}{\partial p} \phi_i & i \neq j \\ -0.5 \phi_j^T \frac{\partial M}{\partial p} \phi_j & i = j \end{cases} \tag{7}$$

N is the total number of mode shapes, p is the parameter of design (moment of inertia, Young's modulus, thickness of the beam, etc.), K and M are the stiffness and mass matrices, respectively. ω_i and ϕ_i are the ith eigenvalue and the associated eigenvector, respectively.

It should be noted that this technique is still a popular method even up it suffers from the problem that to get a precisely sensitivity of the calculated mode shape all the mode shapes are needed and should be integrated.

2.2.2 Nelson's Method

In the same context, Nelson's method, as another powerful method, can be also used to approximate the derivatives of the mode shape to a design variable p. The mode shape sensitivities can be calculated as follows [5]:

$$\frac{\partial \phi_i}{\partial p} = q_i + c_i \phi_i \tag{8}$$

where q_i is a particular solution and $c_i \phi_i$ is the homogeneous solution.

$$c_i = -\phi_i^T M q_i - 0.5 \phi_i^T \frac{\partial M}{\partial p} \phi_i \tag{9}$$

It can be clearly observed that in this approach only the mode of interest is of im-portance to be identified to evaluate exactly the sensitivities, which represents an advantage in comparison to the Modal method in example. Furthermore, in this method, the sensitivity of mode shape to a damage can be defined as a combination between two solutions: particular and homogeneous, for the considered mode. For more details about this method, the reader is referred to [5].

3 Results and Discussions

To apply the presented theory above and assess the efficiency of eigenvector sensitivity and its associated curvature for damage detection and localization in a coupled track-bridge system, the studied system – previously analyzed by Yang et al. [10] – is examined in this section. The key mechanical characteristics of the railway coupled system are gathered in Table 1. The rail and the bridge share the same total length L and are assumed to be S-S However, the assumption of a rigidly supported rail beam does not fully schematize the real behavior of the track, as rails are infinitely long and supported by an elastic foundation composed by rail-pads, sleepers, and ballast (modeled via the Winkler approximation, where the foundation's equivalent force is proportional to the vertical displacement of the beam).

To address this discrepancy, an ''artificially'' length before and after the bridge could be introduced. However, this is omitted here, as its influence is generally deemed negligible [10]. In the context of FE modelling, the track and the bridge beams are discretized into ne=50 elements of equal length le. Only vertical displacements at each node are retained in the calculations, with rotational degrees of freedom neglected, as the focus solely on flexural vibration modes. Regarding damage, it is assumed to affect only the system's stiffness (with mass properties remain unchanged) and is simulated by reducing the Young's modulus of the rail beam element. In the subsequent analysis, the mode shape sensitivity and curvature are as $d\phi/dE = \|d\phi/dE\|^{-1}(d\phi/dE)$and $d\phi/dE_{curvature} = \|d\phi''/dE\|^{-1}(d\phi''/dE)$.

Table 1. Mechanical properties of the track-bridge system [10].

Parameter	Value
Length L(m)	32
Bending stiffness of the rails $EI_r(Nm^2)$	$1.2831.10^7$
Mass per unit length of the rails $\rho A_r(kg/m)$	120
Bending stiffness of the bridge $EI_b(Nm^2)$	33.10^8
Mass per unit length of the bridge $\rho A_b(kg/m)$	4800
Foundation stiffness $k_f(N/m^2)$	33.10^6

3.1 Single Damaged Element

In general, higher-order modes of a structure are more sensitive to damage com-pared to lower-order modes. However, this subsection focuses exclusively on lower modes, as extracting higher-order modes from experimental measurements is not an easy task due to limitations in resolution and noise. By considering a single damage in the upper beam (rails) of the coupled system – specifically, introducing damage at element 5 – and retaining 80 modes of vibration to obtain the first derivatives of the first mode shape using the Modal method. Fig. 3. predicts the mode shape sensitivities and their associated curvatures of the fundamental modes of the rail and the bridge, respectively.

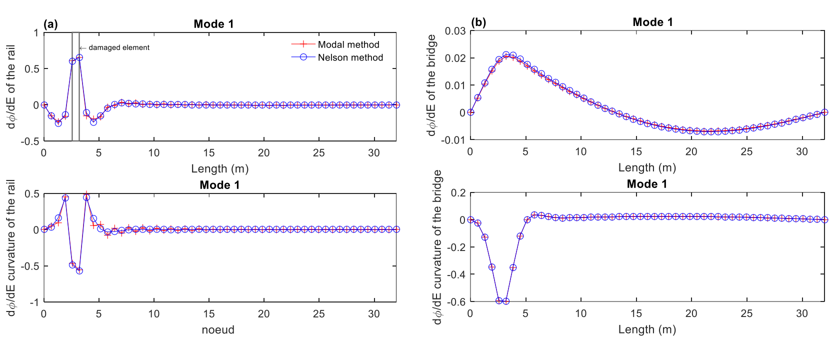

Fig. 3. First mode shape sensitivities and their associated curvatures to a damage in the rail beam; a) rail, b) bridge.

From Fig. 3, two observations emerge. First, there is perfect agreement between the two numerical methods for computing the first eigenvector sensitivity. Notably, Nelson's method achieves reliable results easily, which is an advantage, as experimental scenarios often permit identification of only limited number of modes. Second, for the first derivative of the rail mode, the damaged element (element 5) is readily localized without requiring to curvature calculations. In contrast, the bridge mode does not exhibit this capability. However, the curvature of the sensitivity effectively localizes the damage both for the rail and bridge modes, demonstrating its robustness as an efficient indicator.

3.2 Multiple Damaged Elements

Following the procedure outlined in subsection 3.1, three damaged elements in the rail beam (elements 5, 9, and 14) are considered in this case. It should be noted that while the design parameter in this investigation is the Young's modulus E, other parameters such as, the moment of inertia I, depth of the section h, element length *l*e could be also examined. Aligning with the single damage analysis presented earlier, Fig. 4 depicts the derivatives of the fundamental mode with respect to the Young's modulus of the rail beam element taking into account multiple damage, three damaged elements, are depicted in Fig. 4.

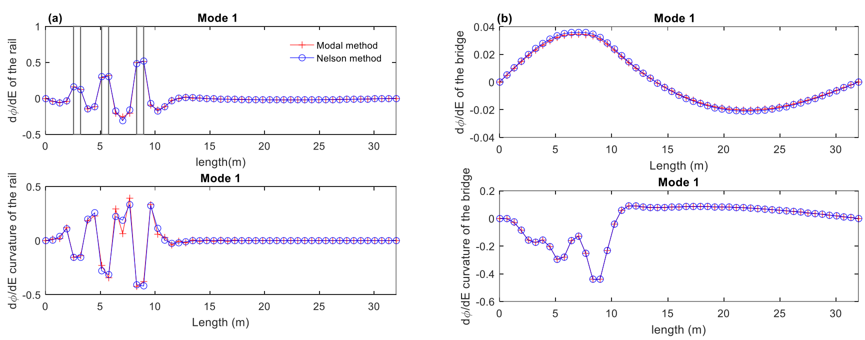

Fig. 4. Mode shape sensitivities and their associated curvatures to a damage in the rail beam; a) rail, b) bridge.

The curves in this figure illustrate the performance of Nelson's method when using fewer modes. They reveal that the mode shape sensitivities of the rail beam are primarily influenced by the presence of damage and can be utilized to localize it with precision. However, for the bridge's mode shape, damage in the rail beam is only intercepted through the curvature analysis. The sensitivity vectors here fail to provide consistent information about the exact damage location – a finding consistent with earlier observations in the single damaged element scenario.

3.3 Rail in Elastic Foundation

In this example, which is adopted from the first case study, the lower beam is omitted and the upper one resting directly on the elastic linear foundation. The studied S-S rail beam is of length L=200m, Young's modulus E=210GPa, quadratic moment I=3055.10^{-5} m^4, and mass per unit length m=120kgm. The beam is divided into 200 elements, each with an elemental length of 1m. Two damage scenarios are analyzed : single damaged element and three damaged elements in the rail beam. Damage is modeled as a reduction of the elastic modulus (Young's modulus E) of the affected elements.

By considering a single damaged element, Fig. 5 illustrates the sensitivity vectors of the first two mode shapes of the rail beam and their associated curvatures for a damaged element (element 2), located near the beam support (critical position in general). For both methods, Nelson's method and the Modal method, the damaged element's position can be identified using either the sensitivity vectors or their curvatures. While the Modal method needs to be

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computed with more than 80 modes for convergence, this result aligns well with the findings in the dual beam case presented earlier. The efficiency of the curvature of the mode shape sensitivity vector in the estimation of the damage's position is thus demonstrated.

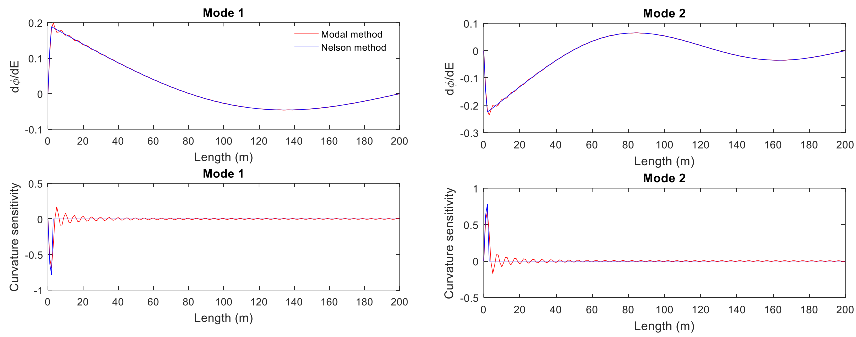

Fig. 5. Rail mode shape sensitivity and its associated curvature to damage for a damaged element: 2.

For the first two mode shapes of the S-S rail beam resting on homogeneous elastic foundation, Fig. 6 reaffirms, another time, the good agreement between the two used methods for computing derivatives of the modal parameter (mode shape). It is also highlights a key limitation: similar to the coupled beam case (discussed earlier), localizing multiple damaged elements (three in this case) using only the sensitivity vector of first mode remains challenging. Notably, in the prior coupled beam analysis, the eigenvector sensitivity of the first mode sufficed to localize damage for three dam-aged elements. However, this example underscores the critical role of the curvature of the sensitivity vector in advancing structural damage detection to the localization level, even without explicit knowledge of damage induced modal changes, particularly within the finite element modelling framework.

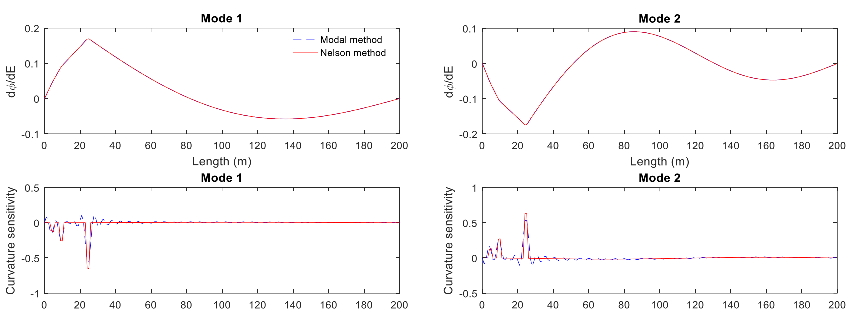

Fig. 6. Sensitivity of the rail mode shape and its associated curvature to damage for damaged elements: 5, 10 & 25.

4 Conclusion

In the present work, a theoretical study is conducted in a view to test the feasibility of the mode shape sensitivities vector and its associated curvatures to localize a single or multiple damage present in a studied railway system. A FE model of the dual-beam system schematizing a ballasted track railway bridge is created, the mode shapes and natural frequencies were extracted, a damage here simulated by a loosing of the elastic modulus E of the beam structure and the stiffness of the ballast foundation. Two methods, Nelson's method and Modal method, are used commonly to approximate the first derivative of the eigenvector information to a design parameter. The obtained results have shown that the first method approximates with low number of modes the sensitivity which is not the case with the second method, also, it is observed that the sensitivity of the first mode can give exact information about the position of the damage. The curvature, additionally, has been observed as an efficient indicator to localize the default, either for single or multiple damaged elements.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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