Statistical Analysis and Probabilistic Characterization of the Mechanical Behavior of ABS: Application of Student's t-Distribution and Weibull Distribution

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Abstract. This study, presents a detailed of a statistical study of how acrylonitrile butadiene styrene (ABS) behaves mechanically when subjected to uniaxial tensile testing. By combining Student's t-distribution with the Weibull distribution, we capture the variability in mechanical properties and build robust reliability models. Our analysis covers both flawless specimens and those containing geometric flaws, such as notches and holes. From the data, we identified crucial statistical parameters, established 90% confidence intervals, and set design guidelines to ensure a 99% probability of survival. Importantly, the study highlights how these geometric defects significantly weaken the material's load capacity. This probabilistic framework equips engineers with valuable insights to optimize ABS component design and reliably predict their performance in real-world applications.

Keywords: ABS – Student's t-distribution – Weibull distribution.

1. Introduction

The use of polymer materials in modern engineering lets a scientistes to understand and assess their reliability. Acrylonitrile Butadiene Styrene (ABS) stands out as a widely used engineering polymer thanks to its strong mechanical performance, ease of processing, and affordability. Polymers like ABS can change how they behave depending on the situation. Because we need them to work well in important uses, we must use advanced statistics to study how they perform. Recent studies highlight that probability

methods are now very useful in material research. For example, tests on 3D-printed ABS parts show that their performance can vary greatly depending on the manufacturing process. For example, in [1], the authors studied how materials respond to different stresses. Later, Wang and al. [2] reviewed Weibull methods for polymer composites to predict material failure using this probabilistic models. These studies show a general change in materials science, moving away from fixed models to using statistical methods, especially for polymers. In this study, we try to better understand how ABS behaves by using two probability methods together: Student's t-distribution and Weibull distribution. Kim and colleagues [3] say this combined way works well to study thermoplastics under different stresses. Our work builds on that by looking closely at both typical behavior and rare extreme cases in ABS. Supporting this approach, Chen et al. [4] demonstrate that combining Student's t-distribution for small-sample analysis with Weibull distribution for failure prediction offers robust confidence intervals and reliability criteria. Our investigation focuses on several key aspects: a systematic evaluation of mechanical properties via standardized uniaxial tensile testing; statistical characterization of both pristine ABS specimens and those with controlled geometric defects, such as notches and perforations; development of confidence intervals and reliability metrics to guide design optimization; quantitative assessment of how defects affect mechanical strength and structural integrity. The researchers in [7, 8] show that using advanced probability distributions helps make better predictions when studying polymers. By blending rigorous statistical analysis with practical engineering considerations.
In the present paper, we further explore the behavior of cracked pressure shells by investigating how linear and nonlinear regimes affect crack propagation. Our goal is to improve modeling and prediction of mechanical behavior in cylindrical pressure shells subjected to loads that threaten their structural integrity. This investigation includes a comparative analysis of simulation methods, examining fracture mechanics parameters, the influence of crack size, and optimizing evaluation and design methodologies under both linear and nonlinear conditions.

2. Mechanical Characterization

This study aims to better understand how acrylonitrile butadiene styrene (ABS) behaves mechanically in order to improve its properties. We performed a detailed statistical analysis on seventeen standard ABS samples. The experimental results from this testing campaign are presented in Figures 1.2 and 1.4, while the key mechanical properties derived from these tests are summarized in Table 1.

Table 1. Mechanical Characteristics of ABS Specimens

Paramètres	**E (GPa)**	**σ_e (MPa)**	**σ_{max} (MPa)**
Average	2,74	35,3	36,31
Standard deviation	0,26	0,23	0,36

The experimental graphs in Figures 2 and 4 show the unique behavior of the polymer tested. The material stretches, low elongation before breaking and limited plastic deformation. The differences in the results come: from various experimental uncertainties; the alignment accuracy of the testing machine and the testing conditions like temperature or humidity. These differences are minor and show the importance of using statistical analysis to understand the material's mechanical properties.

3. Statistical Study

3.1 Student's Distribution

The Student's t-distribution was created by the statistician William E. Gosset. It is an important tool for analyzing data, especially when the sample size is small and the overall population's standard deviation is not known. Shown by the symbol "t," this distribution helps calculate confidence intervals accurately, which makes the results more reliable. In this study, we use this distribution to determine a confidence interval. The chance that the highest stress measurements include the data's average is 90%. Then we have to evaluate our uniaxial tensile tests. The formula for this distribution can be written like:

$$P\left[-t_{(\alpha;n-1)} \leq \frac{X-\mu}{S/\sqrt{n}} \leq t_{(\alpha;n-1)}\right] = 1-\alpha \tag{1}$$

This equation shows the range μ and the true average is:

$$P\left[X - t_{(\alpha;n-1)} \, S/\sqrt{n} \leq \mu \leq X + t_{(\alpha;n-1)} \, S/\sqrt{n}\right] = 1-\alpha \tag{2}$$

Where: X is the average value of the sample; S is the estimated standard deviation of the sample, n is the number of samples. The value t(α; n−1) is the critical number from Student's t-distribution for a confidence level of (1−α) with (n−1) degrees of freedom. Also, we have: X Arithmetic mean of the sample; S Sample standard deviation; n Sample size; t(α;μ) Student's t-coefficient for a given risk and α Risk threshold (significance level). Applying Student's distribution requires first calculating the fundamental statistical parameters. The arithmetic mean is determined using the following formula:

$$X = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3}$$

where x_i represents the value of the i^{th} observation.

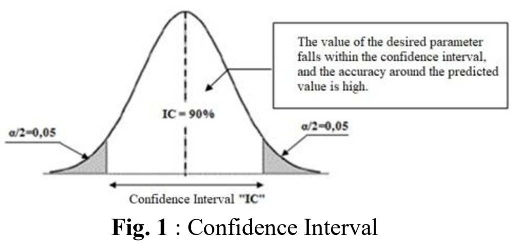

Fig. 1 : Confidence Interval

Application to Tensile Tests: The confidence interval for the true mean was calculated using equation (1.2) based on the results of the static tensile tests. This analysis helped define the upper and lower bounds for the yield strength and ultimate tensile strength of the untested specimens.

Calculation for the Yield Strength: The statistical parameters obtained for the yield strength are: X = 35.30 MPa; S = 0, 236; n = 17 Specimens; α = 0, 1 (Confidence level of 90%); α/2 = 0, 05; t (α/2;n−1) = 1, 746;

$$\mu = 35{,}30 \pm 1{,}746 \times \frac{0{,}236}{\sqrt{17}} \quad (4)$$

Therefore, the confidence interval for the true mean μ of the yield strength is:

$$\mu \in [35.20MPa; 35.40MPa] \quad (5)$$

Calculation for the Maximum Stress: The statistical parameters obtained for the maximum stress are: X = 36.31 MPa; S = 0, 367; n = 17 Specimens; α = 0, 1 (Confidence level of 90%); α/2 = 0, 05; t (α/2;n−1) = 1, 746;

$$\mu = 36{,}31 \pm 1{,}746 \times \frac{0{,}367}{\sqrt{17}} \quad (6)$$

Therefore, the confidence interval for the true mean μ of the maximum stress is:

$$\mu = [36{,}15MPa \pm 36{,}46MPa] \quad (7)$$

The confidence interval represents a range of values that are statistically consistent with the experimental results. In this case, with a 90% confidence level, these intervals provide a reliable range within which the true values of the yield and maximum stresses are expected to fall. Tables 2 and 3 presents the experimental values (within these confidence intervals) showing the consistency and reliability of the measurements.

Table 2. Set of Yield Strength Values within the Confidence Interval

Experiment No.	Yield Strength (MPa)
1	35,34
2	35,22
8	35,20
11	35,30
17	35,33

Table 3: Mechanical Characteristics of ABS Specimens

Experiment No.	Maximum Stress (MPa)
6	36,18
7	36,25
8	36,29
9	36,32
10	36,38
11	36,41

3.2 Weibull Distribution

The Weibull distribution is widely used in the reliability analysis of polymer materials [7] to characterize mechanical properties and their variability [4]. The goal of this analysis is to use the experimental data from tensile tests on ABS specimens to determine the yield strength and maximum stress corresponding to a failure probability of less than 1% [9]. Probabilistic models are based on the following fundamental relationships:

$$P_f = 1 - exp\left[-\left({}^{\sigma}/_{\sigma_o}\right)^m\right] \tag{8}$$

P_m is the failure probability. it is complementary to the survival probability (Ps):

$$P_f + P_s = 1 \tag{9}$$

3.2.1 Methodology for Parameter Determination

The determination of Weibull distribution parameters is based on the graphical method and the analytical method. The first one involves fitting an optimal linear regression (coefficient equal to 1) to the logarithmic relationship [ln(ln(1/Ps)); ln(σ)], allowing for a visual estimation of the parameters. m and σ_0 are then determined by:

$$P_s = 1 - P_f \tag{10}$$

$$P_s \leq 1 \Rightarrow \ln(P_s) \leq 0 \Rightarrow \frac{1}{P_s} \geq 1 \tag{11}$$

$$ln\left(\frac{1}{P_s}\right) = \left(\frac{\sigma}{\sigma_o}\right)^m \tag{12}$$

$$\ln\left[ln\left(\frac{1}{P_s}\right)\right] = m\,ln\left(\frac{\sigma}{\sigma_o}\right) \tag{13}$$

$$\ln\left[ln\left(\frac{1}{P_s}\right)\right] = m(\ln\sigma - \ln\sigma_o) \tag{14}$$

$$\ln\left[ln\left(\frac{1}{P_s}\right)\right] = m\ln\sigma - \text{m}\ln\sigma_o \tag{15}$$

These equations are rewritten in the form:

$$\text{y} = mx + C \tag{16}$$

Where $y = \ln\left[ln\left(\frac{1}{P_s}\right)\right]$, $x = \ln(\sigma)$ and $C = -\text{m}\ln\sigma_o$

3.2.2 Analytical Determination of Weibull Parameters

In Weibull statistical analysis, two key parameters define the distribution: the shape factor m (or the Weibull modulus), and the scale parameter σ_0. It can be determined analytically by using:

$$\sigma_0 = e^{\left(-\frac{C}{m}\right)} \tag{17}$$

Where σ_0 Represents the scale parameter, C is the y-intercept, m is the Weibull modulus and e is the base of the natural logarithm.

3.2.3 Determination of the Critical Stress

The critical stress, σ_c, can be determined by Ps of 1%. This value is obtained from this equation:

$$P_s = e^{-(\sigma/\sigma_o)^m} = 0.1 \quad (18)$$

It can be expressed in terms of the other as:

$$\sigma \to \sigma_s = \sigma_o[-ln(0{,}01)]^{1/m} \quad (19)$$

With σ_0 is the scale parameter and m is the Weibull modulus. This formula enables the direct calculation of the critical stress associated with a 1% survival probability.

3.3 Modeling Using the Weibull Distribution

This distribution is a useful statistical method for describing the chance that specimens will survive when stretched. It helps to connect the stress applied with the strength of the samples.

3.3.1 Fundamental Expression

Ps can be described using the two-parameter Weibull distribution:

$$P_s = e^{\left[-\left(\sigma/\sigma_o\right)^m\right]} \quad (20)$$

σ is the applied mechanical stress and σ0 is the scale parameter characteristic of the material.

3.3.2 Linearization and Parameter Identification

In this subsection, we use a double logarithm transformation to show that the experimental data matches the Weibull distribution and to find its main parameters:

$$ln\left(ln\left(\frac{1}{P_s}\right)\right) = m\ln(\sigma) - m\ ln(\sigma_o) \quad (21)$$

The graphical representation of this expression as a function of ln(σ), figure 2, allows to check if the Weibull model holds by ensuring the data forms a linear relationship. This method also helps identify important parameters: the Weibull modulus m is the slope of the line, and the scale parameter σ0 is found from where the line crosses the y-axis. Using this approach, we can fully describe the statistical spread of mechanical strength in the group of specimens studied.

3.4 Analysis of the Elastic Limit

3.4.1 Determination of Weibull parameters

The elastic limit analysis was performed by plotting the relationship ln[ln(1/Ps)] as a function of ln(σ), as shown in Figure 2. The linear regression of the experimental data resulted in the following equation:

$$y = 140x - 499{,}7 \quad (22)$$

the Weibull modulus is m = 140 and the scaling parameter is σ_0= 35.49 MPa.

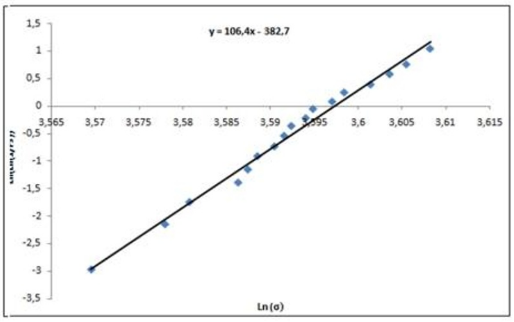

Fig. 2 : Evolution of ln(ln(1/Ps)) as a function of ln(σ) for the Elastic limit

The analysis of Figure 2 reveals an excellent linearity of the experimental point cloud, thus confirming the adequacy of the Weibull distribution to model the statistical behavior of ABS specimens subjected to tensile stresses.

III.4. 2 Probability Analysis

The survival probability distribution (Ps) and failure probability distribution (Pf) as a function of the elastic stress are presented in Figure 3. This representation allows for visualizing the transition between the material's reliability and failure domains. The results obtained demonstrate a strong correlation between the experimental data and the Weibull model.

3.5. Analysis of Maximum Stress

3.5.1 Characterization of the Weibull Distribution

The analysis of the maximum stress was carried out by studying the relationship between ln[ln(1/Ps)] and ln(σ), presented in figure 4. The linear regression of the experimental data led to the following equation:

$$y = 106{,}4x - 382{,}7 \tag{23}$$

The analysis of these results reveals consistency with the Weibull distribution, similar to that observed for the elastic limit. The identified characteristic parameters are m = 106.4 and σ0 = 36.48 MPa.

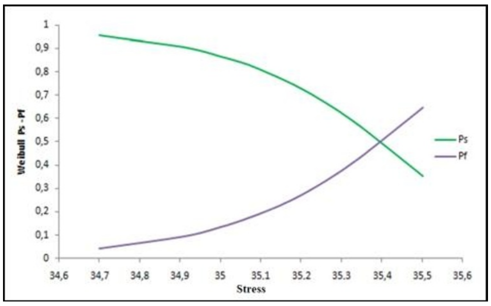

Fig. 3: Probability of survival and failure as a function of elastic stress

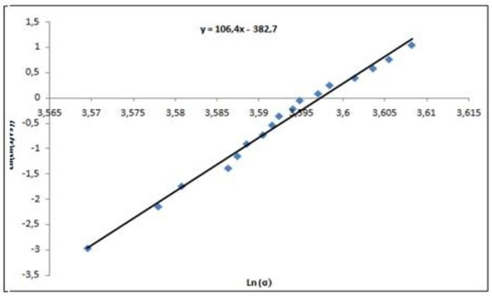

Fig. 4: Trend of Ln (Ln (1/Ps)) as a function of Ln (sigma) for the maximum stress

3.5.2 Probability Distribution

The representation of survival probabilities (Ps) and failure probabilities (Pf) as a function of the maximum stress is illustrated in figure 5.

3.5.3 Determination of the Critical Stress

To establish the maximum allowable stress corresponding to a failure probability of less than 1% (Ps = 0.99), the Weibull equation was solved.

$$P = e^{\left[\left(\frac{\sigma}{\sigma_o}\right)^{m}\right]} = 0.01 \qquad (24)$$

The solution of the equation (23) leads to a critical stress, given by . $\sigma_c = 36{,}89\ MPa$ Note that, this value represents the maximum stress threshold that should not be exceeded to guarantee a 99% reliability of the tested specimens.

III.5. 4 Superposition of the Survival and Failure Probability Curves

Weibull probability diagrams are important tools used to study how materials fail. They help to show different stages of failure risk, like early failures, normal use, and final failure times. By looking at both reliability (Ps) and failure (Pf) probabilities together, we can better assess risks even when we have only a few test samples. This helps make better decisions [2]. When we look at the combined curves from tensile tests on new ABS samples, showing both elastic and maximum stresses (figure 6), it shows that acrylonitrile butadiene styrene is uniform. This analysis helps us find three clear zones with different features:

- **Zone I: [0 – 35.40 MPa] Elastic domain:** Characterized by reversible behavior; Constitutes the operational safety zone and No permanent deformation is observed
- **Zone II: [35.40 – 36.89 MPa] Controlled plastic domain:** Presence of irreversible damage; Controllable plastic deformation and critical transition zone

Zone III: [36.89 – 37.00 MPa]Unstable plastic domain: Characterized by uncontrollable damage; Catastrophic failure zone and complete Structural Failure
This precise delimitation of the mechanical behavior zones allows for the establishment of rigorous design criteria and the optimization of the material's service conditions.

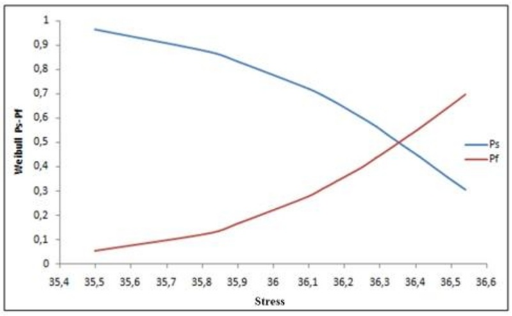

Fig. 5: Survival and Failure Probability as a Function of Maximum Stress

3.6 Behavior of drilled and simply notched specimens: Statistical Analysis

3.6.1 Student's Distribution

A tensile testing campaign was conducted on two series of 17 specimens, each with a specific geometric configuration: a notch with a depth of a1 = 7 mm or a2 = 14 mm, each associated with a circular hole with a diameter of 3 mm. The statistical results are presented in Table 4.

Table 4: Mechanical Characteristics of ABS Specimens

Parameter	**a_1 = 7 mm + Ø**	**a_2 = 14 mm + Ø**
Average (MPa)	20,19	20,19
Standard deviation (MPa)	4,81	4,81

Confidence Intervals: The statistical analysis allows the definition of the following confidence intervals for specimens:with a1=7 mm + Ø, μ ∈ [20, 13; 20, 26] MPa and a2 = 14 mm + Ø, μ ∈ [4.76; 4.87] MPa.

3.6.2 Weibull Distribution

We verified the fit of the experimental results with the Weibull distribution by plotting [ln(1/Ps)] against ln(σ) for both specimen configurations (figure7). The analysis showed excellent linearity of the data points in both cases, confirming that the Weibull model is a good fit.
The survival probabilities (Ps) and failure probabilities (Pf) were determined for both specimen configurations and are shown in figure 8.

Table 5: Weibull Parameters

Weibull Parameters	a_1 = 7 mm + Ø	a_2 = 14 mm + Ø
m	138,00	138,00
σ_0 (MPa)	40,49	40,49

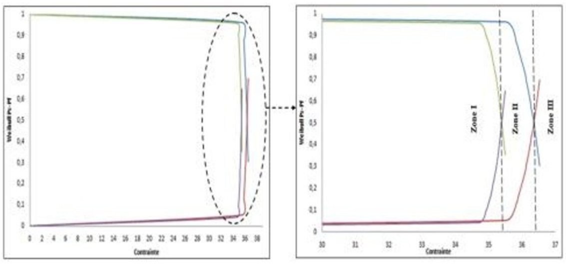

Fig. 6: Superposition of survival probability and failure probability curves for virgin ABS specimens

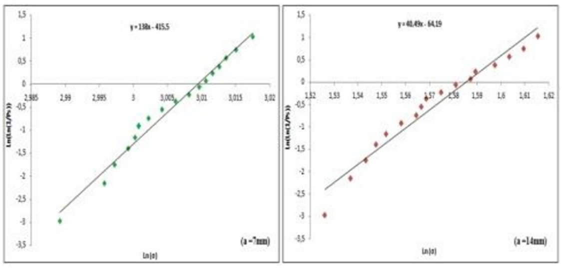

Fig. 7: Trend of the ln(ln(1/Ps)) curve as a function of Ln(sigma) for drilled and notched specimens (a1=7mm+Ø hole and a2=14mm+Ø hole)

3.6.3 Comparative analysis of survival and failure probabilities for drilled and notched specimens

Regarding the conditions, the comparative analysis focused on two distinct geometric configurations: configuration 1: 7 mm notch with a 3 mm diameter hole (a1 = 7 mm + Ø3 mm); configuration 2: 14 mm notch with a 3 mm diameter hole (a2 = 14 mm + Ø3 mm); a 54% reduction for the a1 configuration (7 mm + Ø3 mm); a more pronounced 87% reduction for the a2 configuration (14 mm + Ø3 mm)

4. Results and Analysis:

Figure 8 presents the superposition of the survival and failure probability curves for the two configurations studied. The data analysis reveals a significant decrease in structural reliability. These results highlight the critical influence of defect geometry on the mechanical behavior of ABS polymer. The simultaneous presence of a notch and a hole leads to a substantial change in the material's reliability properties, with a particularly marked effect for notches of larger dimensions. This quantitative analysis establishes a direct correlation between the defect geometry and the degradation of the mechanical performance of ABS polymer, thus providing essential data for the optimization of the design of structural components.

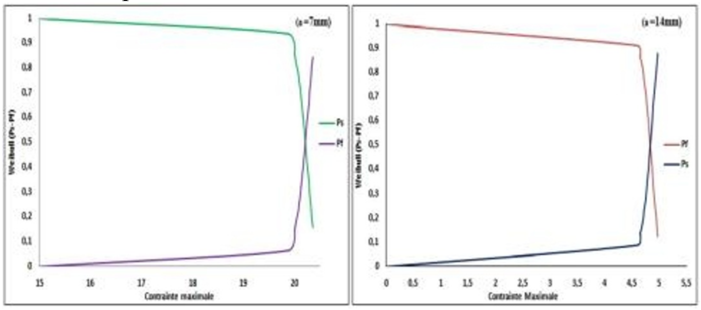

Fig. 8: Survival Probability:Failure Probability Curve for Specimens with Combined Defects

5. Conclusion

This study developed a statistical method to describe the mechanical properties of acrylonitrile butadiene styrene (ABS). By combining Student's t and Weibull distributions, the approach closely matched the experimental data ($R^2 > 0.95$). The detailed study of the mechanical behavior of new specimens showed three separate zones, each with its own type of deformation: first, a linear elastic zone; next, a nonlinear transition zone; and finally, a clear plastic deformation zone. This three-part description helps us better understand how failure happens and gives a solid basis for improving structural designs. In the case of two both notches and holes at the same time, the results show that the material become weak. Note that, in the worst cases, the strength dropping by as much as 87%. Through these results, we can see the importance of considering different defects work together when designing parts. In addition, this data and models of this study give an idea for improving the design of ABS components. These findings also open new paths for studying engineering polymers,

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