A shape factor for particle size distribution caracterization

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Abstract. This paper examines the parameter ν, a shape factor of the particle size distribution (PSD) in granular materials, and its broad applications in geotechnical engineering. Defined as the slope of the PSD in a log-log plot, ν provides a funda-mental measure of distribution shape, capturing the evolution of granular materials under varying conditions. While initially developed to quantify particle break-age, ν has proven to be a versatile tool with applications extending to the modeling of PSD evolution, micromechanical analysis, and the influence of environ-mental factors such as moisture. The paper explores the theoretical foundation of ν, its connections to fractal theory, and its role in describing granular transformations under mechanical and hydraulic influences. Additionally, the relationship between ν and other PSD-based breakage parameters, such as those proposed by Marsal, Hardin, and Einav, is examined, highlighting its advantages as a unifying descriptor of grain size evolution. By synthesizing these perspectives, this study underscores the value of ν as a robust and adaptable parameter for advancing the understanding of granular mechanics and geotechnical engineering.

Keywords: Granular mechanics, particle size distribution, particle breakage, mi-cromechanics, unsaturated soil mechanics.

1 Introduction

Granular materials, such as soils, sands, and rockfills, are fundamental to numerous engineering applications, ranging from foundation design to embankment construction. The mechanical behavior of these materials is inherently linked to their particle size distribution (PSD), which evolves under external loading, environmental conditions, and internal interactions. Understanding and quantifying this evolution is cru-cial for predicting the performance of granular materials in both natural and engineered systems. Among the various parameters proposed to describe PSDs, the shape factor ν has emerged as a particularly effective tool, offering a simple yet comprehensive measure of PSD evolution.

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The parameter ν, defined as the slope of the PSD in a log-log plot, provides a robust framework for characterizing the shape of granular distributions [1]. Initially introduced to quantify particle breakage [2], ν has since been recognized for its broader applications, including describing PSD evolution under varying moisture levels, stress paths, and loading conditions [3]. Its theoretical foundation in fractal theory further enhances its utility, as it aligns with the self-similar nature of granular materials subjected to extreme loading. This paper explores the multifaceted role of ν in granular mechanics, emphasizing its versatility as a shape factor for PSDs.

The concept of ν builds on earlier work by Turcotte [4], who established the fractal nature of fragmentation processes in geological materials. This fractal framework was later adapted by Einav [5], [6] to develop a breakage mechanics theory, which introduced the idea of using PSD evolution to quantify particle breakage. While Einav's approach focused on energy-based breakage indices, the parameter ν offers a simpler alternative by directly linking the slope of the PSD to the extent of break-age.

Recent studies have further expanded the applications of ν beyond breakage quantification. Recent work by Salami and Jaafri [7] utilizes ν as a shape factor to characterize the particle size distribution (PSD) of granular materials, offering a simple yet comprehensive measure of PSD evolution. By analyzing the UNSODA data-base, the study establishes a direct correlation between ν and the shape parameter μ of the soil water retention curve (SWRC). The study analyzes 301 soil samples from the UNSODA database [8], demonstrating that ν effectively captures the influence of PSD on water retention, particularly in soils with lower void ratios. Coarser soils exhibit stronger correlations between ν and μ, while finer soils show more complex behavior due to additional hydraulic mechanisms. The proposed empirical formula provides a practical tool for predicting SWRC based on PSD, offering a balance between simplicity and physical relevance. This work highlights the versatility of ν in advancing the understanding of soil-water interactions and its potential for geotechnical and hydrological applications.

This paper aims to provide a comprehensive review of the parameter ν, focusing on its role as a shape factor for PSDs. We begin by exploring its theoretical foundations and micromechanical implications, followed by a discussion of its applications in quantifying particle breakage and describing PSD evolution in wet and dry conditions. Experimental data from various studies are analyzed to validate the effective-ness of ν, and its relationship with other breakage parameters, such as those proposed by Marsal [9], Hardin [10], and Einav [5], is examined. By synthesizing these insights, this paper underscores the value of ν as a robust and versatile tool for characterizing granular materials in geotechnical engineering.

2 Theoretical framework

The parameter ν is defined as the slope of the linear trendline of the PSD in a log-log plot. Mathematically, it is expressed as:

$$log\ (F_{(d)}) = \nu\ log\ (d/dmax) \tag{1}$$

where F(d) is the cumulative mass fraction of particles with size less than d, and dmax is the maximum particle size. The parameter ν captures the shape of the PSD, with lower values indicating a more spread-out distribution and higher values reflecting a more uniform grading.

The concept of ν is rooted in fractal theory, which suggests that the size distribution of granular materials subjected to extreme loading converges to a fractal distribution. For a fractal distribution, the number of particles N with size greater than d follows a power law:

$$N \, a \, d^{-\nu} \qquad (2)$$

This relationship highlights the self-similar nature of granular materials and provides a theoretical basis for using ν as a descriptor of PSD shape.

3 Applications of the parameter

While ν was initially developed to quantify particle breakage, its applications extend beyond this specific use. Below, we explore the versatility of ν in characterizing granular materials.

3.1 Micromechanical significance

The parameter ν is not merely a macroscopic descriptor of particle size distribution (PSD); it also holds significant micromechanical implications. By assuming a fractal-type distribution as described by equation (2), ν can be directly related to the specific surface area generated during particle fracture. This relationship bridges the gap be-tween the macroscopic evolution of the PSD and the micromechanical processes that govern particle breakage. Specifically, ν captures the scaling behavior of the PSD, which is inherently linked to the energy dissipation mechanisms at the particle scale. As particles fracture under external loading, the increase in surface area is reflected in the evolution of ν, making it a powerful tool for understanding the interplay between macroscopic and microscopic phenomena in granular materials.

At the micromechanical level, the parameter ν is explicitly defined as a function of the specific surface area S by the equation:

$$\nu_{(S)} = 1 + ln_{((S - S_u) / \alpha A + d^{\nu f - 1}) / ln(d)} \qquad (3)$$

Here, S is the specific surface area at a given state, Su the ultimate surface area corresponding the fractal distribution, d is the characteristic particle size and νf the ultimate value of the parameter ν. a and A are scaling parameters related to the shape of the grains and their size distributions, which normalize the surface area increase relative to the material's behavior.

The logarithmic scaling ties the evolution of ν(S) to the material's properties and particle size, making it a powerful tool for analyzing the interplay between energy dissipation mechanisms at the particle scale and the macroscopic evolution of granular materials.

3.2 Quantifying particle breakage

The parameter ν provides a straightforward yet effective measure of the evolution of the particle size distribution (PSD) due to breakage. As particles break, larger grains fragment into smaller ones, leading to a more dispersed PSD and a reduction in ν. Experimental studies have demonstrated a strong correlation between ν and the extent of particle breakage, making it a valuable tool for modeling crushable granular materials.

Unlike traditional breakage parameters, such as those proposed by Marsal [9], Hardin [10], and Einav [5], which require knowledge of the initial and final PSDs, ν can be directly determined from a single PSD. This characteristic makes ν particularly useful when analyzing experimental data where only the current state of the PSD is available.

Additionally, Salami et al. [1] established mathematical relationships linking ν to these conventional breakage parameters:

Marsal's breakage parameter B_M:

$$B_{M} = (v / v_i)^{v/(vi-v)} - (v / v_i)^{vi/(vi-v)} \tag{4}$$

Hardin's breakage parameter B_H:

$$B_H = (v_i - v)/(v_i \cdot (v + 1)) \tag{5}$$

Einav's breakage parameter B_E:

$$B_E = ((v_i - v)/(v + 1)) \cdot ((v_f + 1)/(v_i - v_f)) \tag{6}$$

where v_i is the initial value of v, and v_f represents the ultimate fractal state of the PSD. These formulations allow seamless conversion between different breakage measures and further validate v as an effective parameter for quantifying particle breakage.

3.3 Modelling particle breakage

Konrad and Salami present a framework for modeling particle breakage in granular materials by defining key parameters that describe how the grain size distribution evolves under mechanical loading [2]. Their approach identifies three main stages of breakage: initiation, development, and stabilization. Initially, the granular material remains mostly intact, with little fragmentation occurring. As loading increases be-yond a critical threshold, breakage accelerates, leading to a rapid evolution of the particle size distribution. Eventually, a final stable state is reached, where further loading does not significantly alter the grain structure (Fig. 1.).

A central element of this model is the breakage parameter v, which quantifies the degree of fragmentation by characterizing changes in the slope of the grain size distribution in a log-log plot. This parameter starts at an initial value corresponding to the unbroken state and gradually transitions to a final value representing a fully fractured material. The rate at which this transition occurs depends on both the applied mechanical work and the inherent strength of the particles.

To account for material strength, the framework introduces a parameter that reflects the resistance of grains to breakage. This parameter is linked to the tensile strength of the particles, particularly focusing on the median particle size, where 50% of the grains pass by mass. Empirical correlations suggest that stronger particles exhibit greater resistance to fragmentation, leading to a slower evolution of the break-age parameter.

By integrating these concepts, the model effectively connects particle strength, ap-plied stress, and breakage evolution, providing a comprehensive tool for predicting the behavior of granular materials under loading. This approach is particularly useful in geotechnical applications where understanding the stability of soils and aggregates is crucial for engineering design.
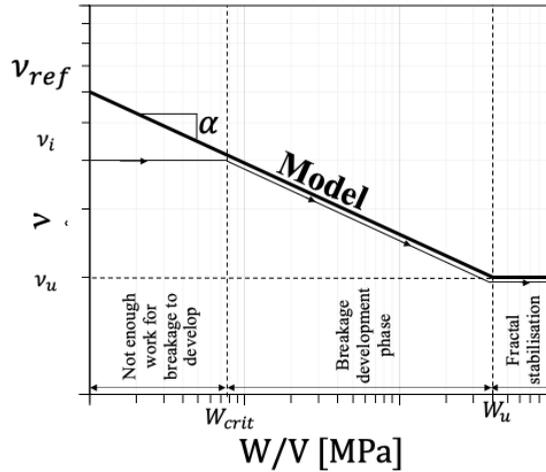
Fig. 1. The evolution of ν according to the model.

3.4 Modelling particle breakage in wet conditions

The effect of moisture on particle breakage is a critical aspect of granular material behavior, as demonstrated by Salami and Konrad [3]. Their study highlights that water weakens particle strength, accelerating the rate of breakage and altering the evolution of the PSD. This effect is attributed to both capillary forces at particle con-tacts and physicochemical weakening of the grains. Experimental results show that when granular materials transition from dry to wet conditions, the initiation of break-age occurs at lower stress levels, leading to increased fragmentation. This behavior is particularly evident in rockfill and other coarse-grained materials, where water infiltration can trigger structural deformations, such as those observed in rockfill dams after reservoir filling or heavy rainfall.

The study further establishes that the evolution of PSD in wet conditions is distinct from dry conditions, requiring modifications to traditional breakage models. Salami and Konrad propose that moisture alters the mechanical work required for breakage, effectively shifting the critical breakage threshold. Experimental results indicate that when water content increases, the parameter controlling the rate of breakage also increases, leading to more rapid PSD evolution [11]. This effect is particularly notice-able when a material is partially saturated before loading, with breakage accelerating once full saturation is reached. The model successfully captures this transition, showing that breakage intensifies beyond a critical moisture threshold, reflecting a funda-mental change in granular mechanics under wet conditions.

A key contribution of this work is the ability to predict breakage evolution under varying moisture levels, particularly when saturation occurs mid-loading. The findings of the study suggest that once a material is saturated, its breakage behavior aligns with that of an initially saturated sample, regardless of when the saturation event occurs. This observation is crucial for geotechnical applications, as it implies that changes in moisture conditions during loading—such as flooding or rapid humidity shifts—can significantly impact granular stability. The proposed framework, validat-ed against experimental data, offers a practical tool for assessing the role of moisture in breakage-prone materials, providing insights into engineering challenges such as embankment stability and foundation performance under fluctuating moisture conditions.

3.5 Correlation with other geotechnical parameters

The soil water retention curve (SWRC) describes the relationship between soil moiture content and matric suction, playing a crucial role in understanding unsaturated soil behavior. It governs key hydraulic and mechanical properties such as permeability, shear strength, and volume change, making it essential for geotechnical applications like slope stability, foundation design, and water infiltration modeling. However, direct experimental determination of SWRC is often time-consuming and costly, requiring specialized equipment and extensive testing. To address this challenge, re-searchers have developed numerous empirical and semi-empirical models to estimate SWRC based on more easily measurable soil properties. Many of these models lever-age the fractal nature of soil pore structures, recognizing that pore size distributions exhibit self-similar scaling patterns [12]. Over the years, various attempts have been made to correlate SWRC with geotechnical parameters such as PSD, porosity, and specific surface area to improve prediction accuracy while reducing experimental effort [13], [14], [15], [16].

In a recent study by Salami and Jaafri [7], the focus is placed on the correlation between the SWRC shape parameter μ and the PSD shape parameter ν, offering a physically meaningful approach to linking soil grading properties with hydraulic behavior. The parameter μ defines the steepness of the SWRC in a log-log plot, where higher values indicate rapid desaturation over a narrow suction range, and lower values correspond to more gradual moisture loss:

$$S_r = (S / S_0)^\mu \qquad (7)$$

Where S_r is the saturation of the soil, s the matric suction, s_0 a reference suction corresponding to a specific saturation (often chosen at air-entry value), and μ is a dimensionless fitting parameter that characterizes the slope of the SWRC on a log-log scale.

Given that pore size distribution strongly influences water retention, it is reasonable to expect a relationship between μ and ν, which captures the grading characteristics of granular materials. Previous studies have suggested that PSD influences SWRC, but a direct mathematical link has remained elusive [17]. This paper seeks to establish such a connection, demonstrating that μ can be predicted using ν, particularly in granular soils where pore structure is largely dictated by particle arrangement.

To analyze this correlation, a comprehensive dataset was extracted from the UNSODA (UNSaturated SOil DAtabase) [8], which contains detailed information on soil hydraulic properties, including SWRC measurements, PSD characteristics, and void ratio data. The database includes a wide range of soil types, from coarse sands to fine-grained clays, allowing for a robust evaluation of the ν-μ relationship across varying textures and porosities. The study specifically focuses on 301 soil samples where both PSD and SWRC data were available, ensuring consistency in parameter estimation. By systematically analyzing the dataset, the work highlights how the strength of the ν-μ correlation varies with soil texture, revealing that well-graded granular materials exhibit the strongest relationships, while finer soils require additional considerations due to aggregation and clay-related effects. By analyzing this dataset, the study identifies a clear trend linking μ and ν, with void ratio e acting as a modifying factor. The proposed correlation is given by:

$$\mu = (0.72.e - 1).\nu \qquad (8)$$

This equation accounts for the influence of soil structure on water retention, where lower void ratios (denser soils) strengthen the correlation, while higher void ratios introduce deviations due to more complex pore structures. The equation provides a practical tool for estimating SWRC from PSD data, particularly in granular soils, offering an efficient alternative to experimental measurements.

The results of the work provide a practical tool for estimating SWRC from PSD data, reducing the need for costly and laborintensive experimental measurements. The proposed empirical formula accounts for void ratio effects, improving prediction accuracy across different soil types. This correlation has significant implications for geotechnical and hydrological applications, enabling more efficient modeling of unsaturated soil behavior in engineering practice. By establishing a direct link between PSD and SWRC shape parameters, the study contributes to a more unified understanding of soil-water interactions, enhancing the predictive capabilities of unsaturated soil mechanics while maintaining a strong physical basis.

4 Conclusions

The parameter ν emerges as a robust and versatile tool for characterizing granular materials in geotechnical engineering. Its theoretical foundation in fractal theory aligns with the self-similar nature of granular materials, making it a comprehensive descriptor of PSD evolution under various conditions. Initially developed to quantify particle breakage, ν has demonstrated its utility in a wide range of applications, including modeling PSD evolution in both dry and wet conditions, understanding mi-cromechanical processes, and correlating with other geotechnical parameters such as the soil water retention curve (SWRC).

Experimental validation from multiple studies underscores the effectiveness of ν in capturing the shape and evolution of PSDs, highlighting its reliability as a measure of particle breakage and granular material behavior. By bridging the gap between macroscopic PSD evolution and micromechanical processes, ν provides a unified frame-work for understanding the interplay between particle fracture, energy dissipation, and material properties.

Furthermore, the relationship between ν and established breakage parameters, such as those proposed by Marsal, Hardin, and Einav, enhances its applicability in both theoretical and practical contexts. Its ability to describe PSD evolution in un-saturated soils and its correlation with hydraulic properties further extend its relevance to geotechnical and hydrological applications.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Salami, Y., Konrad, J.-M. & Jaafri, R. Quantifying particle breakage through a shape factor of the particle size distribution. Geomechanics and Geoengineering 1–10 (2024).
2. Konrad, J. M. & Salami, Y. Particle breakage in granular materials — A conceptual framework. Canadian Geotechnical Journal 55, 710–719 (2018).
3. Salami, Y. & Konrad, J. Modeling Effects of Moisture on Particle Breakage. International Journal of Geomechanics 21, 04021157 (2021).
4. Turcotte, D. L. Fractals and fragmentation. Journal of Geophysical Research 91, 1921–1926 (1986).

5. Einav, I. Breakage mechanics—Part I: Theory. Journal of the Mechanics and Physics of Solids 55, 1274–1297 (2007).

6. Einav, I. Breakage mechanics-Part II: Modelling granular materials. Journal of the Mechanics and Physics of Solids 55, 1298–1320 (2007).

7. Salami, Y. & Jaafri, R. Correlation Between Shape Parameters of Grain Size Distribution and Soil Water Retention Curves. International Journal of Geo-Engineering (2025).

8. Nemes, A., Schaap, M. G., Leij, F. J. & WoÈsten, J. H. M. Description of the unsaturated soil hydraulic database UNSODA version 2.0. Journal of Hydrology 251, 151–162 (2001).

9. Marsal, R. Large-scale testing of rockfill materials. Journal of Soil Mechanics and Foundations Division, ASCE 93, 27–43 (1967).

10. Hardin, B. O. Crushing of Soil Particles. Journal of Geotechnical Engineering 111, 1177–1192 (1985).

11. Salami, Y., Dano, C. & Hicher, P.-Y. A Multiscale Investigation of the Effects of Water on Grain Crushing. Geotech Geol Eng 40, 5895–5903 (2022).

12. Tyler, S. W. & Wheatcraft, S. W. Application of Fractal Mathematics to Soil Water Retention Estimation. Soil Science Society of America Journal 53, 987–996 (1989).

13. Arya, L. M. & Paris, J. F. A Physicoempirical Model to Predict the Soil Moisture Characteristic from Particle-Size Distribution and Bulk Density Data. Soil Science Society of America Journal 45, 1023–1030 (1981).

14. Aubertin, M., Mbonimpa, M., Bussière, B. & Chapuis, R. P. A model to predict the water retention curve from basic geotechnical properties. Canadian Geotechnical Journal 40, 1104–1122 (2003).

15. Fredlund, M. D., Fredlund, D. G. & Wilson, G. W. Prediction of the Soil-Water Characteristic Curve from Grain-Size Distribution and Volume-Mass Properties. in (Rio de Janeiro, Brazil, 1997).

16. Schaap, M. G., Leij, F. J. & Van Genuchten, M. Th. rosetta : a computer program for estimating soil hydraulic parameters with hierarchical pedotransfer functions. Journal of Hydrology 251, 163–176 (2001).

17. Russell, A. R. How water retention in fractal soils depends on particle and pore sizes, shapes, volumes and surface areas. Géotechnique 64, 379–390 (2014).